A LITTLE BOOK ON MAP PROJECTION

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BY

MARY ADAMS

LONDON
GEORGE PHILIP & SON, Ltd., 32, Fleet Street
Liverpool: PHILIP, SON, & NEPHEW, Ltd., South Castle Street
1914

PREFATORY NOTE

BY

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PROFESSOR OF EDUCATION IN THE UNIVERSITY OF LONDON

No kind of apparatus is more generally used in schools than maps. It is therefore essential that the teacher should understand how maps are made. Too often pupils accept maps merely as a part of the nature of things. Occasionally, they observe that the shape of a country or a continent varies from map to map, and the more thoughtful among them appeal to the teacher for an explanation. Accordingly map-projection ought to be included in the training college course. As a matter of fact it formed part of my course as a student in training, and I have a vivid memory of the difficulty of the subject as presented to me in a certain crabbed text-book.

College would have been a pleasanter place for me and my fellow students had we been able to use Miss Adams' little treatise. She begins at the beginning: she takes time to present her matter: she uses all manner of concrete illustrations: she appeals constantly to the reader's experience: she does not shirk difficulties but she reduces them. Her approach is psychological. She is more concerned to present facts in such a way as to meet the pupils' needs than she is to maintain the logical symmetry of the subject matter. Her work is accordingly of a special value to professional teachers, and I can confidently recommend it to my fellow workers in the Training Colleges.

It may not be out of place to mention that, though we bear the same surname, Miss Adams and I are in no way related. I have never met her, and know nothing of her except what I have learned by reading her book.

JOHN ADAMS.

AUTHOR'S PREFACE

This little book is intended for the use of pupils in Secondary Schools, Central Schools, Higher Elementary Schools, and the Senior classes of those ordinary Elementary Schools in which special attention is given to practical geography. Apart from the Appendix, except in one paragraph which is not essential, no knowledge is assumed of even the Trigonometrical ratios. In the Appendix a few examples are given of the application of higher mathematics to map projection.

The writer is indebted for much information to the work on Map Projections by Arthur R. Hinks, M.A., F.R.S. (Cambridge University Press, 1912). The student who desires to calculate map projections other than those of the simplest type, or to estimate the errors of scale or form in any map, should study Mr. Hinks' book. The object of the present writer is description rather than calculation and, except for the sake of illustration in the Appendix, only the simplest calculations have been used.

A special feature is that all the projections, except Fig. 15, are drawn to the scale of a globe two inches in diameter, which differs from the scale of 1 : 250,000,000 by only one quarter per cent. The projections are, therefore, directly comparable one with another.

The reader is advised to draw the projections contained in this book, or parts of them, to the scale of the school globe and to compare the meridians and parallels with those on the globe for different latitudes and longitudes.

A few illustrations have been introduced from mechanics, physics, and geometry, which may appear to have but a very remote connection with geography, but they will afford subjects for lessons which will appeal to many pupils, and lead to an increased interest in map projection. It is very undesirable that the subjects of school study should be separated by "water-tight bulkheads."

M. A.

Forest Hotel,
Vermala-sur-Sierre,
1st January, 1914.

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APPENDIX

A LITTLE BOOK ON MAP PROJECTION

THE MAKING OF MAPS

How to draw a Straight Line

Few people have thought how difficult it is to draw a straight line when no straight-edge is available. If you draw a straight line by means of a straight-edge you are only reproducing or copying a straight line which somebody else has made. The problem is to draw a straight line when there is no straight line to copy. The carpenter often lets a stretched string draw it for him. A string when stretched tries to make itself as short as possible, and as a straight line is the shortest line that can be drawn between its ends, a stretched string, if its weight is supported, will lie in a straight line, so the carpenter chalks his string, carefully fixes the two ends, stretching the string forcibly, and then he lifts the middle of the string an inch or two above the log on which the line is to be drawn, and lets go. The string, released, falls back into the straight line and hits the board a blow which shakes off some of the chalk and leaves a straight chalk line for the sawyer.

If you are prepared to take plenty of time in making a straight-edge you can make two at once by clamping together two thin boards and planing the edge. As they are planed together they will be either both straight and your job will be complete, or they will be both convex or both concave. They must be both alike. If you unclamp the two boards and bring the planed edges together to try to make them fit as for a joint

—if they fit exactly for their whole length they will both be straight. If they are both concave (hollow) they will touch at the ends and show an open space in the middle (Fig. 1, A). If they are both convex (rounded) and touch in the middle they will gape at each end (Fig. 1, B). They may be both partly rounded and partly hollow, as in Fig. 1, C. In any of these cases they must be clamped together again and replaned until they fit exactly along their whole length.

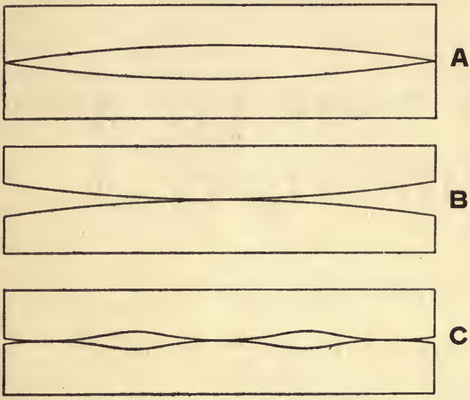


Fig. 1.

If you are content to make your straight line on a piece of paper, the easiest way is simply to fold the paper into a distinct crease. When the paper is opened out again the two "leaves" fit along the crease necessarily, while when the paper is folded the crease is a common edge to the two leaves, so the result is the same as that obtained by planing the two pieces of wood until they fit exactly when the edges are brought face to face.

It is very difficult to draw straight lines of very great lengths. A straight line only a few hundred yards in length is not easy to construct. For very long straight lines, as in gunnery practice and surveying, *sight lines* are taken, that is to say, use is made of the fact that light travels in space, or in air which is throughout at the same temperature and pressure, in straight lines. If there are three points, A, B, and C, and B just appears to coincide with C when looked at by an eye at A, then A, B, and C are in a straight line. This is the principle of sighting a gun and of using the telescope for astronomical measurements. In making maps, directions, which are straight lines, are found by looking at the distant object, the direction of which from the point of view we want to know, through a telescope and moving the telescope until the image of the small

object seen in the telescope is on a mark fixed in the telescope in the centre of the field of view. When this is the case the mark, the centre of the object-glass of the telescope and the distant object are in one straight line. Graduated scales on the mounting of the telescope enable us to learn the direction of the line joining the fixed mark in the telescope and the centre of the object-glass. This direction is the direction of the distant object as seen by the eye.

How to make a Plane Surface

While a line has length only, a surface has length and breadth. Among surfaces a plane surface is one on which straight lines can be drawn through any point in any direction upon the plane. If you apply a straight-edge to any part of a table-top you can turn the straight-edge round completely and it will exactly coincide with the surface in all directions if the table-top is a true plane. Engineers employ test planes of cast iron which are known as surface plates. While the straight-edge can be made to test a plane surface a surface known to be plane can be equally well employed to test a straight-edge, and in the engineer's shop the surface plate is the foundation of all accurate workmanship.

It was an interesting problem, solved in practice by Sir Joseph Whitworth, to make a plane surface without any plane being available by which to test it. A very much greater degree of accuracy was required than could be obtained by a straight-edge applied to the surface in different directions. No straight-edge existed as accurate as the planes required. The method adopted was to make three planes and test them against one another two and two. The cast-iron surfaces having been made as truly plane as the ordinary machine tools rendered possible, were scraped by hand tools and rubbed together from time to time with a little very fine red lead between them. Where they touched the red lead was rubbed off, and then the plates were scraped again to remove the little elevations thus revealed, and the process continued until all the projecting points were removed. If

only two plates were worked together one might be convex (rounded) and the other concave (hollow), and if they had the same curvature they might touch all over and yet not be plane; but if three plates, A, B, and C, are worked together and if A fits both B and C, and A is concave, B and C must both be convex and will not fit one another. If B and C both fit A and also fit one another at all points, then all three must be truly plane.

When once an accurate surface plate has been made, others can be made one at a time and tested by laying them on the standard plate and moving them over its surface with a very little red lead between. When two surface plates made as truly plane as possible are placed gently on one another without any red lead, the upper plate will float almost without friction on the very thin layer of air which takes a very long time to escape from between the plates because they are everywhere so very near together.

Developable Surfaces

There are a great many surfaces which are not plane, but which, if made in thin material like paper or thin sheet metal, can be unrolled or "developed" into plane surfaces without stretching in any direction. A piece of thin tube forming a cylinder if cut from end to end can be laid out flat, or unbent, so as to form a plane surface. This is equally true whether the pipe be circular in section or of any other shape. The pipe is a cylinder, and a straight line can be ruled through any point on its surface so as to be parallel to its axis, but if a straight-edge is applied to the surface in any other direction it will not coincide but will touch the surface at one point only. When a cylinder is unrolled or developed the figure formed is a rectangle of breadth equal to the circumference of the cylinder.

A cone is another surface which can be unrolled into a plane surface if it is cut from its vertex or point down to its base. The base may be circular or of any other shape, but the surface can be equally well unrolled into a plane. A straight-edge can always be laid on the surface of a cone so as to lie in contact

with the surface from the vertex to any point in the base, but if the straight-edge be applied to the cone in any other direction than that which passes through the vertex it will touch the cone only at one point. The cone, like the cylinder, is a curved surface which, by unrolling or simple bending, can be laid out flat. All surfaces which admit of being unrolled into planes are called developable surfaces, and through any point in such a surface it is always possible to draw a straight line upon the surface in one direction. The unrolling to form the plane takes place at right angles to this straight line. Figs. 2 and 2A show a cone in elevation and its development. The cone is supposed to be cut along the central generating line in the first figure. It is not true that every surface on which straight lines can be ruled can be unrolled into a plane.

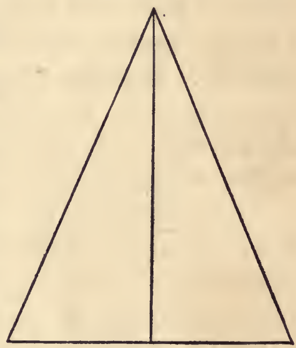


FIG. 2.

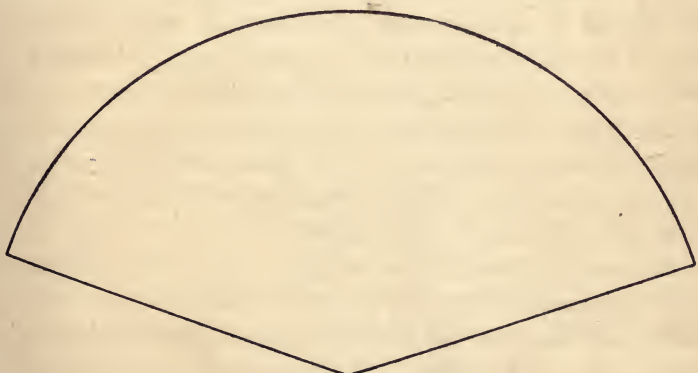


FIG. 2A.

UNDEVELOPABLE SURFACES

The great majority of curved surfaces cannot be unrolled into planes and cannot be made by rolling up plane sheets of thin material whether paper or sheet metal. It is obvious to

every one that a sheet of paper can be rolled into a tube or cylinder, and most people are familiar with the process of rolling a square sheet of paper into a conical bag with one corner as vertex of the cone. If, instead of using a square sheet of paper the paper be cut into a quarter circle with the corner as centre, the base of the cone will be accurately circular instead of irregular as in the case of the square sheet rolled into a conical bag. Any other sector can be rolled into a cone the sloping side of which is the radius of the sector and the circumference of the base the arc of the sector. You cannot, however, by any contrivance fit a sheet of paper to the surface of a round ball, nor can you accurately fit the paper to any portion of the ball without stretching it. If you thoroughly wet the paper you may be able to press it into close contact with a considerable part of the surface of the ball, but if you let the paper dry on the surface of the ball and then remove it you will find that it does not become flat, but retains the curvature of the ball where it has been in contact. The paper has been permanently stretched where it has dried in contact with the ball. To make a sheet of metal fit the ball you would have to hammer it as silversmiths or coppersmiths hammer a sheet of metal to "raise" it into the form of a cup or bowl. A sphere then differs from a cone or cylinder in the essential feature that its surface cannot be unrolled into a plane, and a plane surface cannot be bent or rolled to fit the sphere without stretching it in certain directions. The sphere, along with the great majority of curved surfaces, is undevelopable.

The greater part of the work of metal plate workers who work in tin plate, sheet zinc, or galvanised iron consists of the making of surfaces which are made up of planes, cylinders, and cones or parts of cones. All such work can be shaped by simply bending the plates after cutting them to the proper shape, that is, the shape of the development of the surface. The bottom of a tin kettle is usually flat; the sides are conical and can be formed by simply bending a strip of tin plate cut to the proper shape, but the top of the kettle and the lid are generally dished so as to be convex in all directions. This requires that the metal should be stretched under the hammer,

and the metal so "raised" forms a surface which is undevelopable.

Maps

Maps are invariably printed on plane surfaces. They may be kept rolled up on a roller, but they are always "developed" or laid out flat for use. They are intended to represent portions of the earth's surface, sometimes the whole of the surface. But the shape of the earth is nearly that of a ball, and no portion of its surface can be developed or unbent so as to form a plane surface like a map. Setting aside the inequalities in the earth's surface, such as mountains and valleys which are represented on the maps merely by symbols, and taking the surface of the ocean when smooth, no portion is plane and the curvature is quite apparent in a distance of three or four miles, so that in a map of such a very small country as England we have to represent on a plane a portion of the earth's surface which is bent through 5 or 6 degrees. A point at sea-level in the middle of England stands about four miles above the straight line joining Berwick-on-Tweed with St. Alban's Head.

The Figure of the Earth

A "Terrestrial Globe" is the only accurate representation of the earth's surface. It may be made to any scale, and all the Continents and Islands and their divisions shown upon it will be reduced to the same scale. The great international map of the world now being prepared is to show countries on the scale of 1 to 1,000,000 or about 15·71 miles to the inch. A globe about 41 feet 9⅓ inches in diameter would approximately represent the earth on this scale. The earth is not quite a sphere, and a more accurate representation would make the equatorial diameter about 7926·6 miles and the polar diameter about 7900·1 miles; but we do not propose to take into account the departure of the earth's surface from the truly spherical form. We cannot get an accurate representation of any large portion of a spherical surface on a map, and it would be useless at this stage to complicate the question by attempting to make

corrections for the flattening of the earth's surface at the poles. The metre was originally intended to be equal to one ten-millionth of the quadrant of a meridian from the equator to the pole. The length of the meridian was not known precisely when the metre was determined, but on this basis the quadrant of the meridian of our globe would be 10 metres and the diameter therefore $\frac{40}{\pi}$ metres, or 12·732 metres. Taking the metre to be 39·37 inches, this is equivalent to 41 feet 9·27 inches. We will suppose that we have constructed the model globe 41 feet $9\frac{1}{3}$ inches in diameter, and that there has been properly drawn upon it all the land areas with the principal physical features, political divisions, towns, etc. Our task is to consider how the figures on the surface of this globe can be most usefully copied on paper knowing that an accurate copy is impossible.

Elementary Principles of Map Projection

If the surface of the globe were developable, in order to get our map we should only have to bend a piece of tracing paper over the surface and trace upon it the details we required. The tracing paper when removed and laid flat would then be a true map, and could be reproduced by any of the methods adopted by printers. But we cannot make our tracing paper fit the globe, and so some system has to be adopted for transferring, according to an understood rule, the outlines from the globe directly on to a plane surface, which becomes the map, or else on to some developable surface, a cone or cylinder, which forms the map when spread out flat. This transference from the globe to the developable surface is called map projection. The map always differs from the corresponding portion of the globe's surface. When the area mapped is a very small portion of the whole globe the difference may be very slight; when the area is only a few square miles the errors are too small to be detected, and in making a plan of a town or a parish no one troubles about the curvature of the earth. All the measurements are laid down as though the earth were flat. In a map

of England the projection may be so chosen that the differences between the map and the actual surface of the globe are of very little importance, but in making a map of a large continent like Asia, it is impossible to avoid considerable discrepancies between the map and the surface of the globe. The map sheet will not even approximately lie on the globe's surface. The extent and character of the errors in the map will depend on the method of projection adopted, and in order to interpret the map and to obtain from it accurate distances or directions it is necessary to know what method of projection has been employed. Names have been given to all the ordinary methods of projection and there should be printed on every map the name of the projection by which it is obtained. In the same atlas maps will be found based on several different methods of projection, but, except in the case of certain maps of the whole world, the name of the projection is scarcely ever given. The result is that two different atlases or wall maps may show considerable differences in the shape of Asia, Africa, or North America, and no explanation of the differences is apparent. The names of the projections if printed on the maps would explain the differences, and would also give a key to the corrections which ought to be applied to any part of the map. The importance to the navigator of being able to get his directions and distances correctly is sufficiently obvious, and all sea charts are made on a system which enables this to be done.

A plane sheet of paper can be made to touch a globe at any one point, and it is therefore possible to project a map which shall be accurate just at this point of contact, and shall exhibit no sensible error for some distance around it, but the farther we go from the point of contact the farther will our sheet of paper stand away from the globe, and the greater will be the error caused by any method of "projecting" the surface of the globe upon the paper. If we like to bend the paper into a cone or cylinder we can bring it into close contact with the globe along a line, and all along this line we can get an accurate reproduction of the surface of the globe because the paper and the globe touch each other. Very near this line the paper is very close to the globe, and the error produced by any system

of projection is very small, but as we recede from the line of actual contact the error increases. The character and extent of the error will depend on the method of "projection."

Equal-Area Projections

We may choose our method of projection according to the purpose for which the map is chiefly required. There are methods which can even be applied to a map of the whole world, by which the area of every tract of country is accurately represented at the expense of altering the shape of the country, sometimes past recognition if it happens to lie a long way from the central lines (*i.e.* the lines drawn east and west and north and south through the centre) of the map. These projections are called equal-area or "homolographic," sometimes, but wrongly, spelled "homalographic," projections. If you want to give a correct idea of the extent of the British Empire as compared with the possessions of other European Powers you can use a map of the world drawn on one of the equal-area projections and colour the British Empire red. If, as is commonly the case, the British possessions are coloured red on a map which is not based on an equal-area projection, Canada and New Zealand may appear very much exaggerated, and the Russian Empire will also loom large through the magnification of Siberia and the northern part of Russia.

Orthomorphic Projections

In another type of map projection the correct representation of areas is sacrificed in order to preserve the shapes of small tracts of country. In these maps the "scale" is different at different points of the map, generally increasing as we go further away from the equator or the centre of the map, but it is the same in all directions at any one point. With this type of projection large tracts of land such as continents do not preserve their proper shape, but every square mile of land remains almost exactly square, though a square mile may be much larger in one part of the map than in another. These maps cannot be said to be drawn to any particular scale, except at

the equator or at the centre of the map. Such projections are called "orthomorphic" or right-shaped projections. We shall return to this mode of projection after describing some of the methods adopted for the more common maps.

Geometrical Projection

It would be well here perhaps to refer to the meaning of the term "projection" in geometry. Speaking generally, a shadow is a projection, but as a rule it shows only the outline and mass form. If a screen is placed so that the sun is shining upon it perpendicularly, and an object is placed a little in front of the screen, the outline of the shadow is the orthographic projection of the object because the rays of light which produce it are approximately parallel and perpendicular to the screen. The hazy outline of the shadow is due to the apparent size of the sun's disk, so that the rays of light coming from one end of a diameter of the disk make an angle of half a degree with the rays from the other end of the diameter. It is on this account that in order to get a sharp outline the object must be near the screen. If the sun appeared as a bright point the outline of the shadow would be perfectly sharp. Shadows of this kind can be obtained from naked electric arc lights placed a few yards away from the object to be shadowed. If the object is a skeleton made of wire the shadow will be a complete projection showing all the details.

If instead of using the sun an electric light be employed and placed at a distance from the object not very great as compared with the object itself, the rays of light proceeding from the electric arc to the various points of the object will make considerable angles with one another while they all diverge from a common point. They resemble the straight lines which can be drawn on the surface of a cone, all of which meet in the vertex, and the shadow so cast by diverging light resembles a perspective drawing.

If the object is transparent but differently coloured and made of thin material, the shadow, instead of being black, will show the colour of the corresponding parts of the object. A magic-

lantern slide held a few inches in front of a small electric arc light and a few feet away from a screen would show a coloured picture on the screen which is really a shadow picture. If the electric arc could be made indefinitely small and yet bright enough to give the requisite light the shadow picture would be perfectly defined and would be erect on the screen. It is because we cannot get all the light we want from a point and have to use a source of light of appreciable area that we are obliged to use the magic-lantern "projection" lens, which inverts the picture and reverses it right and left, in order to get a sharply defined or properly focussed picture. The projection lens picks up all the light that comes from one point of the picture slide and which falls on the lens and converges it to a point on the screen, so that all the light which reaches any point of the screen comes from a corresponding point of the slide. Thus a perfectly defined and sharp image is produced on the screen for every point on the slide, and the whole picture on the screen is not a shadow but an image of the picture on the slide. As every ray of light which passes through the optical centre of the projection lens proceeds in a perfectly straight line from the slide to the screen, and rays of light from the same point of the slide falling on other parts of the lens are bent to meet the central line of light on the screen and so to form the brightly illuminated image of the point on the slide, it follows that the point on the slide, the optical centre of the projection lens, and the image of the point on the screen are always in a straight line. Thus the magic-lantern picture is an example of perspective projection, the centre of the lens, instead of the source of light, being the vertex of the cone. The reader will see at once that the picture formed must be upside down and will be magnified in proportion as the distance of the screen from the centre of the lens is greater than the distance of the slide from the same centre. If the picture be 3 inches in diameter and 10 inches from the centre of the lens and the screen be 30 feet in front of the lens, the picture will be magnified 36 times, and will be 9 feet in diameter. The slide is a very little further from the centre of the lens than the focal length of the lens, and consequently the magnification is very nearly

equal to the distance of the screen from the lens divided by the focal length of the lens. This enables the proper lens to be selected when the size of the screen and the distance of the lantern stand from the screen are known. If P be the diameter of the required picture in inches, S that of the picture on the slide, D the distance of the screen from the lens, and F the focal length, then $F = \frac{D \times S}{P}$. This discussion of the magic lantern is a digression, but it has some bearing on the teaching of geography.

If the shadow of a wire frame representing the outlines of a solid object is projected by divergent light proceeding from a point, lines nearer the source of light are magnified in the shadow more than those more remote, and the shadow is a "perspective" of the outline of the solid.

Shadow Maps formed by Geometrical Projection

We will now forget all about the magic lantern and its projection lens (the "condenser" is used only to throw as much light as possible on the slide), and we will think only of the shadow pictures which are not true images because there is no concentration of light on the points of the picture, as no lens is used. We will suppose that we have succeeded in getting a powerful source of light so small that it may be treated as a point. An electric arc is good enough if we are working on a large scale and do not want very fine detail. When our source of light is a point that point, instead of the centre of the magic-lantern lens, is the vertex of the cone of rays.

Suppose now that starting with our globe with a diameter one-millionth of the actual diameter of the earth, we get a thin sheet of glass made with exactly the right curvature to fit the surface of the globe and sufficiently large to cover the portion of the globe of which we want to make a map. This is not a practical method of map-making. No one has ever adopted it and no one ever will. It is introduced only for the purpose of explaining methods of projection. Let the glass cover plate be laid on the proper portion of the globe and with transparent

colours such as those used for coloured magic-lantern slides, let the countries, rivers, towns, etc., be traced on the glass. The picture on the glass dome is then a perfectly accurate representation, as far as it goes, of the corresponding portion of the earth's surface on the scale of 1 to 1,000,000. But it is not flat, and therefore is not a map. It is still part of a globe, and we have as yet done nothing towards representing on a plane the details of the surface of a sphere. We are now going to project our transparent picture on to a flat screen, and then we shall have a *map*. Place the thin glass dome so that the middle point of its surface touches the centre of the screen. We will suppose the screen to be vertical, and we will always place our source of light opposite the centre of the screen, that is on the horizontal straight line drawn at right angles to the screen from the point of contact of the glass dome. Now, when a spherical ball rests on a horizontal table the centre of the ball is always vertically above the point of contact, or the vertical line through the point of contact is a diameter of the sphere. Similarly, the line through the point of contact of the dome and screen at right angles to the screen will be a diameter of the sphere of which the dome is a part. The centre of the sphere will be 20 feet $10\frac{2}{3}$ inches from the screen, and the farther end of the diameter will be 41 feet $9\frac{1}{3}$ inches from the screen. Now place the light anywhere in this straight line and a map will be projected on the screen. If the lamp is 20 feet $10\frac{2}{3}$ inches away from the screen, that is, at the centre of the sphere of which the glass dome is a part, the map is called a gnomonic, or central, projection. As the planes of all great circles pass through the centre of the sphere, and therefore through the source of light, all great circles which can be drawn on the dome will be shadowed on the screen as straight lines, for as the source of light is in the plane of each circle the sheet of light proceeding from the source to the points of the great circle is a plane sheet, and the intersection of this plane sheet of light by the plane screen must be a straight line, for two planes always meet in a straight line. This makes the gnomonic projection very useful to navigators because they can draw the great circles along which they wish to sail as straight lines on their maps. This

form of projection is commonly called "gnomonic" because the problem of drawing the meridians on the map is the same as that of drawing the shadow lines of the gnomon on the plane of a sundial so as to graduate the dial.

Now move the light further away from the screen. The map will get smaller as the rays from the source of light to the circumference of the dome become less divergent. The centre portion of the map will remain almost unchanged because the screen is very close indeed to the dome near the point of contact, but the peripheral portions of the map will contract, and this will be more apparent the larger the dome, that is, the larger the tract of country represented on the map. When the source of light has been removed to a distance of 41 feet $9\frac{1}{3}$ inches from the screen it is at the opposite end of the diameter of the sphere of which the dome forms a part. The map on the screen is then called the stereographic projection. The light may now be moved still further away and the map will continue to contract. At a distance of 57 feet $1\frac{1}{3}$ inches it is 1·367 times the diameter distant from the screen, and the map obtained is Sir Henry James' projection, the merits of which will be pointed out later on.

Orthographic Projection

The light may now be moved to a very great distance. The picture on the screen will be very faintly illuminated, but that does not concern us; we are interested only in its shape. If the light were removed so far, say a few miles, that all the rays reaching the dome were practically parallel, or if the light from a star reflected by a mirror so as to be horizontal could be used and the picture on the screen could be seen, the map would be an orthographic projection of the earth's surface.

It will be easily seen that this method could be employed, if only the dome of glass could be made and could bear handling, with a dome covering a whole hemisphere or one-half of the globe, but it could not be employed for a larger portion of the earth's area because any portion of the second hemisphere would overlap a part of the first hemisphere in the shadow picture of the orthographic projection.

Limits and Errors of Gnomonic Projection

It is also worth while to notice that in the gnomonic projection, that is, with the source of light at the centre of the sphere, if an attempt were made to shadow the whole hemisphere the rays of light from the source to the circumference of the base of the dome would all be parallel to the screen and go off to infinity without ever reaching the screen, while the shadows of parts of the dome near the base of the hemisphere would fall on the screen, if it were big enough to receive them, only at an enormous distance from the centre, and it is clear that a map obtained by this process can be useful only for a comparatively small part of the earth's surface ; and then the scale gets larger as we move away from the centre of the map.

There are two reasons for this increase of scale in the gnomonic projection as we go farther from the centre of the map: the screen is further away from the dome and the light strikes the screen more obliquely, making the shadows longer just as shadows lengthen as the sun goes down. On the other hand, in the orthographic projection the rays of light are always at right angles to the screen and circles drawn on the dome parallel to the screen are all projected true to size, but along radii drawn from the centre of the map to the circumference the scale diminishes as we go outwards because the surface of the dome makes smaller and smaller angles with the rays of light and so becomes "foreshortened" in the shadow picture. If a whole hemisphere is projected in this way the narrow belt adjoining the base of the hemisphere is foreshortened indefinitely. The stereographic projection and Sir Henry James' projection are intermediate between the gnomonic in which the scale increases outwards and the orthographic in which it diminishes outwards.

Figs. 24, 25, and 27, on pp. 69, 71, and 74, show gnomonic, stereographic, and orthographic projections of the portion of the same hemisphere comprising all latitudes from 30°, and Fig. 28, p. 75, shows a non-geometrical projection of the same area in which the correct distances have been preserved along the meridians. As all these figures are drawn so that the

central portion of each map is on the same scale they may be directly compared with one another. In the gnomonic projection, as we recede from the centre of the map the scale increases both along the parallels of latitude and along the meridians, but more rapidly along the meridians than along the parallels. In the stereographic projection both scales are increased as we recede from the centre, but in exactly the same proportion so that geographical features retain their *shape* though their *size* is increased towards the circumference of the map. This gives the projection its orthomorphic character. In the orthographic projection the parallels of latitude appear of their true length, but the meridian distances are diminished towards the circumference, so that the projection is neither equal-area nor orthomorphic, but it differs from orthomorphism in exactly the opposite way to the gnomonic projection. In the last projection, illustrated in Fig. 28, the meridian scale is preserved unchanged, as the scale along the parallels is preserved in the orthographic projection, but the scale along the parallels is, in this case, increased as we recede from the centre of the map. Meridians and parallels and latitude and longitude are described on pp. 26–31.

Geometrical Projection on a Cylinder

If the screen is made of flexible material it may be bent into a cylinder of the same radius as the dome. It will then touch the dome all along a circular line, and then there is no need to restrict the dome to a hemisphere or smaller area. A complete glass sphere may be employed and the light placed at the centre. The shadow map will then be thrown upon the cylinder and will be exact to scale along the whole length of the circle of contact, so that we get a map which is correct along the whole length of a line drawn right across it instead of being correct only in the neighbourhood of the central point as is the case when the screen is flat. To make our map we must draw, in permanent colour, the shadow picture thrown by the light on the cylindrical screen, and then lay the screen out flat. The highest and lowest points of the sphere will lie on the axis of the cylinder, and the line passing through these points and the

centre of the sphere will never meet the surface of the cylinder, and rays of light in these directions will pass on to infinity. The shadows of points near the highest and lowest points of the sphere will fall on the cylinder at a very great distance from the circle of contact. In the figure the shadow of the little circle *ab* is the circle AB of the same diameter as the sphere itself. A circle, however small, drawn around the point P will produce a shadow circle of the same diameter as the cylinder if the cylinder is long enough for the shadow to reach it at all. It is clear, therefore, that by this simple method of projection the scale of the map is true only in the neighbourhood of the circle of contact, and as we approach the highest and lowest points of the sphere the map becomes more and more magnified. The magnification, too, is even greater in the vertical than in the horizontal direction, so that while sizes are increased on approaching the poles, shapes are altered by being stretched more in the up and down direction than horizontally. The system is analogous to the gnomonic projection on a plane, and such a system of projection is of very little use except near the circle of contact. It should be noted that this projection is *not* Mercator's.

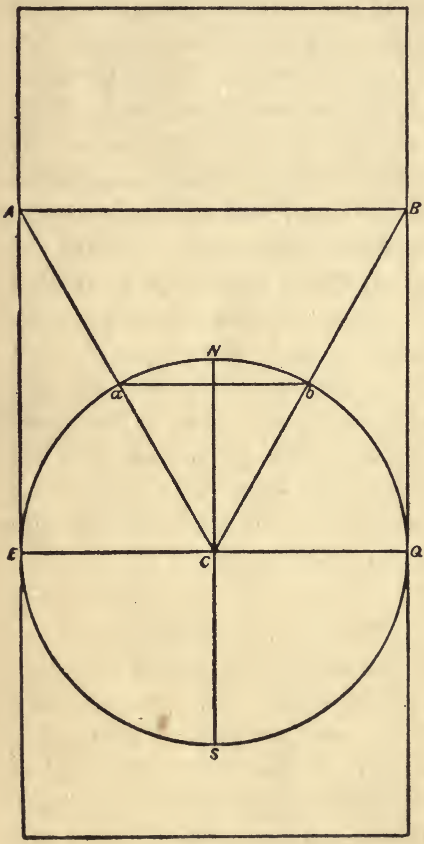


FIG. 3.

Projection on a Tangent Cone

But we have learned that a flexible plane surface can be bent into a cone as well as a cylinder, and the angle of the cone may be made as great or small as we please so that the cone may be so flat as to be nearly a plane, or so steep that any part of it differs very little from a cylinder. We have also learned that any cone if cut from the vertex to the base can be developed or laid flat so as to form a plane surface. In the case of a right circular cone the shape of the development will be the sector of a circle with the vertex of the cone as centre. In order to make our shadow map let us bend our screen so as to form a conical cap, and fit it on to the globe, as in Fig. 4. The cone will touch the globe along a circle, and if we throw the shadow of the painted globe on the cone by means of a light at the centre of the globe the circle of contact will exactly correspond on the globe and cone. The little circle *ab*, which is parallel to the circle of contact, will have for its shadow the larger circle AB, but the increase in size will not be so great as in the case of the cylindrical screen. The shadow of the highest point P of the sphere will be the vertex of the cone, and it will not, as in the case of the cylinder, go off to infinity. Moreover, it is clear from the figure that the line VK is much longer than the arc PK, so that as we go away from the circle of contact, where the shadow and the substance correspond, the scale increases not only horizontally, but also along the sloping sides or generating

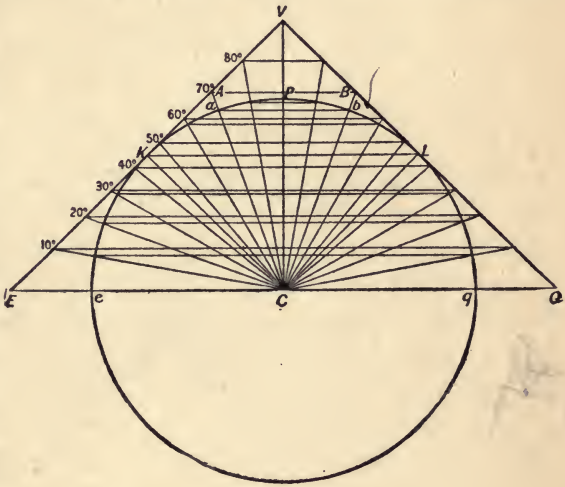


Fig. 4.

lines of the cone. When the cone is laid out flat all the shadow circles like KL and AB will be circular arcs with V, the shadow of P, as centre. (See Fig. 5 in which the pole is indicated by 90°.)

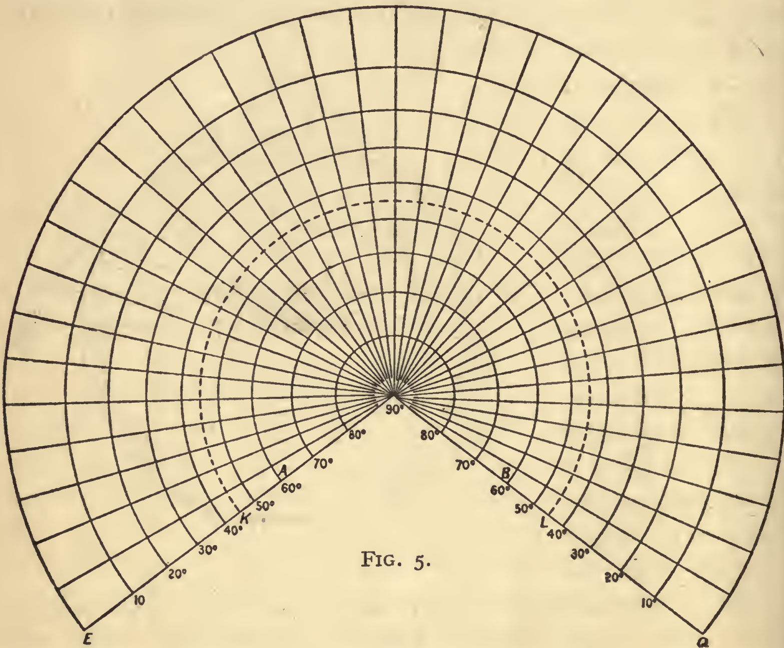


FIG. 5.

By increasing the angle of the cone we can make the circle of contact of the cap and globe come nearer and nearer to P, and by diminishing the angle we can make the circle of contact approach nearer and nearer to EQ, which is the circle of contact for the cylindrical screen.

NON-GEOMETRICAL PROJECTIONS

Thus far we have been dealing with purely geometrical projections, and our shadow maps are true projections of the painting on the globe, being "thrown forward" by the light on to the screen. But in map making the word "projection" is borrowed, and used in a very greatly extended sense. Of the systems of map projection which have been found useful, about fifteen in all, very few are true geometrical projections like our

shadow maps. The gnomonic and stereographic projections are, perhaps, the most important of the true projections, using the word in its geometrical sense. Before attempting to explain the use of the word "projection" in connection with map making, it is desirable to give a little attention to the geometry of a sphere and to the use of latitude and longitude for determining positions on the earth's surface and on maps.

The Earth's Rotation

The earth is continually spinning about a line through its centre, and the points where this line meets the surface are called respectively the North and South Poles. If you were standing on one of the poles you would turn round in the course of the day, always keeping your head pointed to the same point of the heavens (very near the pole star if you stood on the North Pole). You would make a complete turn in about twenty-three hours fifty-six minutes, and assuming it to be summer, so that you can see the sun (and you would not choose to visit the pole in the winter), if you started facing the sun you would be directly facing the sun again in twenty-four hours, or four minutes more than you required to make one complete turn, because in consequence of the earth's motion round the sun, the sun appears to move among the stars in the same direction as that in which you are turning, and you have to turn for the additional four minutes in order to catch him up. It is this spinning of the earth about its polar axis which keeps the direction of that axis approximately steady in space and pointing nearly to the pole star. The rifling of a gun makes the shot spin round many times in a second, and this keeps the shot always moving with its point forwards. If the shot turned over in the air it would meet with much greater resistance, and the resistance would be so distributed on its surface as to deflect it from its true aim. Moreover, if the shot struck sideways instead of "point-blank" the blow would be ineffectual. This is the reason why guns are "rifled" so as to make the shots spin. If you have played with quoits you will know that the iron rings are kept approximately horizontal so as to fall over

the pin by being made to spin in their own plane in the act of throwing. It is true that in consequence of the attraction of the sun and moon the earth's axis of rotation itself describes a cone in space in the course of about 22,000 years, but this change of direction is so slow that the movement is almost imperceptible during the life of any of us, and for our present purpose we need not take it into account.

Physical Reason for the Figure of the Earth

If a fly-wheel is made to turn it tends to become bigger in diameter and by spinning it sufficiently rapidly any fly-wheel can be made to fly to pieces in its attempts to expand. In the early days of electric lighting great trouble was found in keeping together the armature, built up of iron disks and copper wire, in consequence of this bursting tendency when spinning at high speeds. If the load were suddenly taken off a factory steam engine and the governor failed to act the engine would "race," and there would be great danger of the fly-wheel flying to pieces and breaking through the walls and roof of the engine room. If a train were to run at a speed of 500 miles an hour all the steel tyres would burst by their own centrifugal force, and this is only half the speed of the earth's surface at the equator due to the "diurnal rotation." A steel ring placed round the earth at the equator, if it were not attracted (by its weight) to the earth's centre, would fly to pieces through the "centrifugal force" unless it could sustain a pull of 100 tons' weight to the square inch of section. A steel piano wire might just bear this stress, but ordinary engineers' steel will bear little more than one quarter of this pull, and consequently can be made to turn at only one half the linear speed, for the pull increases with the square of the speed.

The surface of the earth (we do not know much about its inside) does not hold together like a steel band. Why, then, does the earth not fly to pieces when it spins? It is because of the "attraction of gravitation" by which every particle attracts every other particle, and so every stone is attracted towards the centre of the earth by all the rest of the earth.

This pull is very much greater than the tendency of the stone to leave the surface in consequence of its circular motion about the polar axis, and this is the case even when the stone is moving at something over 1000 miles an hour as at the equator. A steel belt of a square inch section stretched around the equator would weigh about 200,000 tons, so that although its motion would tend to cause it to stretch and require a pull of 100 tons in the belt to prevent it stretching and bursting, the earth will be pulling the belt down against the ground with a total force of 200,000 tons on the whole ring, and so the belt will lie at rest on the ground, its weight being only slightly diminished by the rotation. The effect of the earth's rotation is to diminish the weight of a pound at the equator by about 24 grains only. If the ring of steel were rotating round the earth with a velocity of 4·9 miles a second or making a complete rotation in 1 hour 24 minutes 40 seconds, it would just rise off the ground.

While, therefore, the attraction of gravitation is far more than sufficient to hold the earth together, the effect is to make any body weigh less at the equator than at the poles.

Suppose the earth were quite soft and accurately spherical and made to rotate as it actually does. The material along the polar axis would weigh more and press with greater force at the earth's centre than a corresponding cylinder in the plane of the equator, so that the pressure at the centre would not balance. The soft material in the direction of the polar axis would be drawn inwards towards the centre and push out the material between the centre and the equator until the equator had expanded so much that the greater length of material between the centre and the equator balanced the greater weight of the material between the centre and the poles, so that the pressure at the centre was the same in all directions. The form of the earth would then be a figure of equilibrium. This result is produced in the earth itself where the equatorial radius or distance of the equator from the centre is about 13¼ miles greater than the polar radius. The equatorial diameter of the earth is about 7926·6 miles, the polar diameter 7900·1 miles. It will be seen that the difference is very little more than one

part in three hundred, and it does not at all affect the principles of map construction. It is easy to take into account the bulging at the equator and the flattening at the poles when the system of map projection has been settled, but for most maps the earth may be treated as a sphere without sensible error. In the Appendix will be found the variation in the length of a degree of the meridian in different latitudes owing to the flattening of the earth.

Great and Small Circles

If you take a well-made ball, like a billiard ball (but it would be better to use a ball of a less expensive kind), and file or grind a flat surface upon it, you will find that the shape of the " flat " is an exact circle. As you continue to file, always being careful to balance your file so as to keep the surface plane, the size of the circle will increase, but it will always be a circle, and the increase will continue until you have filed away half the ball and reached its centre. The piece you have left is a hemisphere, and if you continue the filing the circle will get smaller instead of larger, until you have filed away the whole ball. If you can fix the ball in sufficiently strong cement to hold it against the file the last you will see of the ball will be a little circular spangle. For our present purpose it will serve equally well if, instead of destroying the ball, you take an iron cup or bowl somewhat larger than the ball—a wooden box will do or any other vessel provided the top edge is accurately plane —and fill it with clay or plasticine, or other similar material. Founders' sand, which binds well, will do instead of clay or plasticine. Fill the bowl with the material and by means of a steel rule with a bevelled straight-edge cut the surface so that it is accurately plane and level with the edge of the bowl (or box). Now press the ball a little way into the plastic material or sand and with the sharp straight-edge carefully remove the ridge of material which rises round the ball so as to get the exposed surface perfectly flat again. Then raise the ball. If clay is used and the ball shows a tendency to stick, it may be lightly oiled. You will find that the boundary of the hollow

left by the ball is a circle. If you repeat the experiment, pressing the ball further and further into the material each time, the circle will become larger and larger until just half the ball is below the surface, when the diameter of the circle will be the same as that of the ball. If you carry on the experiment beyond this stage you will not be able to draw the ball out without spoiling the mould, but if you close the material in upon the ball when it is more than half buried you will see that the edge of the material where it touches the ball is still circular, though the circle gets smaller as the ball is pressed farther into the material. For this experiment a good ball may be used, as it need not be injured.

From the observations just made you can learn—

(1) *That every plane section of a sphere is a circle.*

(2) *That the section through the centre is the largest circle that can be cut from a sphere.* It is called a "great circle," while all other plane sections are "small circles" of the sphere.

(3) *That the small circles get smaller and smaller as the plane of section moves away from the centre of the sphere.*

Tangent Planes

In any sphere a small circle may be made as small as we please by removing the cutting plane away from the centre until at last the cutting plane ceases to cut, and becomes the tangent plane at the point of contact. This is the case when a hard ball stands on a plane table, but if you could examine the state of affairs sufficiently accurately you would find that the weight of the ball flattened the surface of the ball and indented the table slightly so that there is contact over a very small circle.

Properties of Great Circles

Suppose that you have a hemispherical cup which exactly fits one half of your ball. Such a cup is formed when the ball has been pressed into the clay or plasticine as far as its centre, but it would be preferable to have the cup made in some harder material. Place the ball in the cup and draw a line round the ball level with the edge of the cup. This line is a

great circle, for its plane passes through the centre of the ball. Mark two points at random on the surface of the ball. Turn the ball in the cup until one of the points is on the edge of the cup, then, keeping this point stationary against the edge, turn the ball about it until the other point comes just level with the edge. Then draw on the ball the great circle passing through the edge of the cup. This great circle will pass through both the points which were taken at random. You infer from this that—

A great circle can be drawn through any two points on the surface of a sphere.

Remove the sphere from the cup and stretch a piece of elastic between the two points. In order to get rid of any effect of friction lift the elastic at different points a little away from the ball and let it fly back. When the elastic has settled down into the position which it likes best it will coincide with the great circle you have drawn, as long as you have not stretched it round the sphere in the direction in which the length of the arc is more than a semicircle. An elastic string when stretched tries to make itself as short as it can. We have seen that on a plane it will make a straight line. On a sphere it lies on a great circle. We therefore infer that—

The great circle is the shortest line that can be drawn on the surface of a sphere between any two points.

This is why sailors like to sail on the great circle between two ocean ports. Generally the compass course has to be continually changed in order to keep on the great circle, but the saving of time and, in the case of steamships, the saving of fuel compensates for the additional trouble.

With a little care great circles can be drawn with considerable accuracy by bending a card so as to touch the sphere all along one of its edges and moving the card so that the edge passes through the two points through which the great circle is to be drawn.

How to Draw Meridians and Parallels on a Ball

On the rim of the cup which fits the ball mark two points exactly opposite to one another. This is not so easy as it

appears when there is nothing to mark the centre of the cup. You can measure the diameter of the cup with a rule and you can draw a circle of this diameter with a compass on a card. The centre will then be marked on the card by the fixed leg of the compass and with a straight-edge you can draw a diameter through the centre. You can cut out the circle and fit it just inside the rim of the cup. The ends of the diameter drawn on the card then mark the two points you want on the edge of the cup. With a triangular file, or other suitable tool, make a little notch at each point on the edge of the cup. Now remove the card and insert the sphere. Mark the two points on the surface of the sphere which correspond to the notches. These points will be at opposite ends of a diameter of the sphere, and the line joining these two points we are going to take as our "polar axis" corresponding to the line about which the earth rotates. One of the points will be the North Pole, the other the South. It does not matter which is north. Now insert a small pin at each pole of the ball, and let the pins rest in the notches in the edge of the cup. We can then make the ball turn in the cup about the polar axis, the pins resting in the notches, steadying the ball and preventing it from turning in any other way. For this experiment the ball should be made of wood or plaster or some composition into which the pins can be driven, and on which we can easily draw lines with a sharp pencil or engrave them with a steel point.

For the purposes of measurement a circle is divided in this country into 360 equal parts, and these are called degrees of arc. If radii be drawn from the centre to each point of division the angles between each radius and the next are all equal, and are simply called degrees and are marked °. For exact measurements the degree is divided into 60 minutes, marked ′, and the minute into 60 seconds, marked ″, so that there are 1,296,000 seconds in a complete rotation. We do not propose to draw 180 circles on our little ball. It will be enough to draw circles at intervals of ten degrees. You have probably used a "protractor" for setting off angles. Protractors generally enable you to mark off single degrees. We

want now to make marks on the edge of our cup for each ten degrees around the circumference, starting from one of the notches. To do this we will return to the card on which the centre is marked, and by means of the protractor draw diameters across the card for every 10 degrees. There will be 18 diameters meeting the circumference in 36 points. Now remove the ball from the cup and insert the card, making a notch on the edge of the cup for every division on the circumference of the card along one semicircle from Pole to Pole.

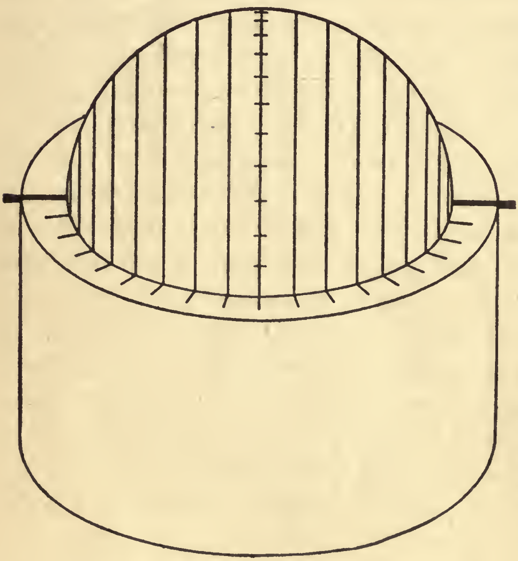


FIG. 6.

It is not necessary to divide the other half of the edge. The semi-circumference of the cup will then be divided into 18 equal spaces, each corresponding to 10° of arc. Now replace the ball, and with the pencil or graver resting in each notch in succession and lightly pressing on the ball, turn the ball completely round. When the pencil is in the notch halfway between the two poles it will draw a great circle. This we will call the equator. It is the great circle, the plane of which is perpendicular to the polar axis. Placed in the other notches the pencil will draw eight circles on each side of the equator. These are small circles, and are called parallels of latitude for their planes are all parallel to one another and perpendicular to the polar axis. The parallels of latitude we will mark in degrees from the equator to the poles. The equator will be marked 0°, and the consecutive parallels 10°, 20°, 30°, 40°, 50°, 60°, 70°, and 80°. The poles themselves will correspond

to 90°. Those on the same side of the equator as the pole we have called North will be parallels of North Latitude and those on the other side parallels of South Latitude.

Now remove one of the pins and turn the ball so that the other pin is at the highest point. The equator will then correspond to the edge of the cup. Make a mark on the ball against each notch on the cup's edge. All these marks will lie on the equator, and will divide one half of the equator into 18 intervals, each of 10°. Now reinsert the pin in the pole of the ball and replace the ball in its bearings. Turn the ball so that the first division on the equator is just on the level of the edge of the cup, and draw a circle right round the ball against the cup's edge. It will pass through both the poles and will cut the equator at two points opposite to each other. Now turn the ball till the next division on the equator is level with the edge and draw another circle, and so on till eighteen circles have been drawn. If we now turn to the last, or nineteenth, mark on the equator we shall find that the first circle drawn has passed through it, and the system of circles is complete at intervals of 10° all round the equator. These circles are called meridians. They all pass through the poles and cut the equator and all the parallels of latitude at right angles, for the planes of the equator and of these parallels are all vertical when the ball is in the cup and the pins on their bearings, while the edge of the cup along which the meridians are drawn is horizontal.

One of the meridians, any one may be chosen, is marked 0°, and the meridians on each side are marked 10°, 20°, and so on up to 180°, which is the other half of the first meridian. If the sphere be placed with the North Pole at the top the meridians to the right as we face the globe are called East, 10° E., 20° E., as the case may be, and those to the left are called West, and correspondingly marked 10° W., 20° W., and so on. It will be seen that while one half of a meridian is 10° E., the other half of the same meridian is 170° W. Similarly the meridian of 20° E. is part of the same circle as 160° W., and so on, the other half of the meridian being that which is on the other side of the poles.

Meridians and Parallels on the Earth's Surface

Our little ball, with the two sets of circles, the meridians and the parallels of latitude, drawn upon it, we will now take as our model of the earth, on which we will suppose corresponding circles to be drawn. Where it is a question only of supposing the circles to be drawn, and not of actually drawing them, it will cost no extra effort to suppose them drawn and numbered for every degree, or every minute or even every second, but no one would attempt actually to draw them on a model globe for intervals of less than a degree. On the earth itself a second of latitude corresponds to a distance of about 102 feet. For the purpose of studying the principles of map projection it is quite enough to suppose that the circles are drawn at intervals of 10°.

It was convenient in drawing the meridians and parallels by the method described to place the polar axis horizontal so that the ball might rest in the cup by its own weight. Henceforth, however, we shall suppose the ball to be turned so that its polar axis is vertical with the North Pole upwards. The equator and all the parallels of latitude will then be horizontal, and the direction of rotation correspondingly to the actual rotation of the earth will carry the face of the ball at which we are looking from left to right, that is from West to East, according to the manner in which we have marked the meridians. As the earth turns from West to East a person on its surface, unconscious of its movement and looking at heavenly bodies, naturally thinks that they are moving from East to West. The sun rises in the East, and sets in the West.

Use of Latitude and Longitude

With the system of meridians and parallels, supposed to be drawn as closely together as we please, we can find our way all over the earth's surface. If we are told that a place is on the meridian which we have marked 30° E., and on the parallel of 40° N., we have only to look for that meridian along the equator, and when we have found it to travel up it until

we reach the parallel marked 40° N., and we have found our destination. In this case 40° N. is the latitude and 30° E. is the longitude of the place.

Fig. 7 shows in orthographic projection the globe with its meridians and parallels for every 10°.

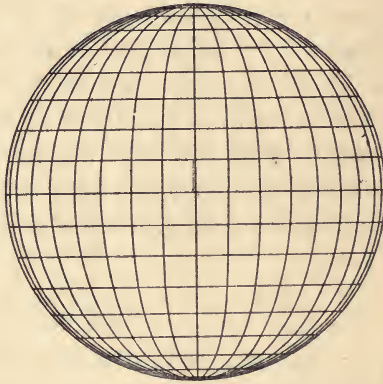


FIG. 7.—ORTHOGRAPHIC PROJECTION OF A GLOBE.

A coast survey means the finding of the latitude and longitude of prominent points along the coast and filling in details by triangulation or otherwise. It is beyond the purpose of this little book to discuss the best means of finding the latitude and longitude of any place. It is sufficient to say that the latitude can be found from the greatest height of the sun or of a known star above the horizon, and the longitude from the difference of time between the instant when the sun is on the meridian of the place, that is apparent noon at the place, and the instant when he is on the meridian at Greenwich. Apparent noon may be found by taking the sun's height in the morning and observing the instant when he is at the same height in the afternoon. The ship's chronometer keeps Greenwich mean time, and although twelve o'clock at noon by Greenwich mean time corresponds only four times a year to the sun being due South as seen from Greenwich, the corrections required for every other day in the year, amounting at most to only a few minutes (16m. 21s.), are all given in the Nautical Almanac for the year and can be allowed for. We shall therefore assume that the latitude and longitude of any place can be correctly determined by an observer at that place.

ALTITUDE AND AZIMUTH

It may be useful here to point out that the angle with the horizon made by the straight line drawn from the observer

to a heavenly body, a mountain top or other elevated object, is called the *altitude* of the object. The angle measured on the horizontal plane between a given direction, generally north, and the horizontal direction of an object is called the *azimuth* of the object. (By horizontal direction of an object which is not on the horizon is meant the direction of the line of intersection of the horizontal plane and the vertical plane through the object and the observer. This makes it possible to speak of the azimuth of an object whatever its altitude, short of the zenith.) The azimuth of an object is identical with its compass bearing from true north, but is measured in degrees, minutes and seconds instead of points.

Trigonometrical Surveys

In a country which is accessible throughout its length and breadth, it is only necessary to determine the latitude and longitude of one point and the direction of one line and then a trigonometrical survey will enable the whole country to be planned, starting from that point, and to be drawn with all the detail required on the globe. To make a trigonometrical survey a line AB has to be drawn from the starting-point A, and its length very carefully measured, and its direction (azimuth) determined. Then, if C be a third point visible from both A and B, and the angle BAC as well as ABC be measured, the sides AC and BC can be calculated, or the point C can be found on a scale plan on which the line AB has been drawn by simply making the angles at A and B correspond to the measured angles. Then BC can be taken as a base line, and the position of any point D, which is visible from both C and B, can be laid down in the same way. The point E can similarly

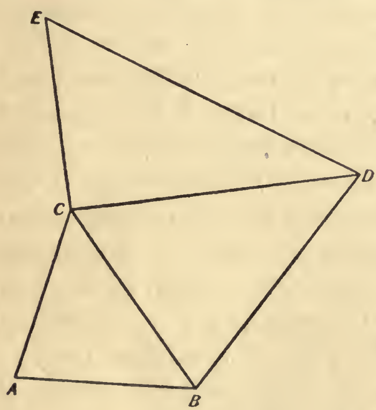


Fig. 8.

be determined from CD, and then by setting up marks all over the country or using existing marks such as church spires, the whole country can be covered with triangles up to the coast, and a series of marks along the coast line will form the extreme points of the outside ring of triangles. When the main features of the country have thus been accurately plotted the details can be filled in within each triangle by triangulation on a smaller scale or by direct measurements, for if the distance of any point from two other points is known the position of the point itself can be laid down. In the Ordnance Survey of the United Kingdom there were 250 Principal Trigonometrical stations, the average length of the sides of the triangles being 35·4 miles. Six base lines were measured, of which one was on Salisbury Plain, about seven miles in length and another about eight miles long at Lough Foyle, 360 miles away. The length of the latter line was afterwards calculated through the triangles from the Salisbury Plain base as a test of the measurements, and the error was found to be little more than 5 inches.

Astronomical Surveys

But long before it is possible to make a trigonometrical survey of a newly-discovered tract of land, it is important that its coast line should be inserted in the world maps. This is done by determining the latitude and longitude of the principal features of the coast and making such observations of the coast line between as will enable it to be drawn approximately when the main features have been properly marked on the globe or map. Portions of the coast line of the Antarctic Continent have already been mapped, but the position of the greater part is at present unknown.

The Problem of Map Projection

It appears, then, that if we have the meridians and parallels of latitude properly drawn on any system of map projection the outline of a continent or island can be sketched in from information given by the surveyors respecting the latitude and

longitude of the principal capes, inlets, or other features, and the character of the coast between them. Copies of maps are commonly made in schools upon blank forms on which the meridians and parallels have been drawn, and these, like squared paper, give great assistance to the freehand copyist. As the meridians and parallels can be drawn as closely together as we please, we can get as many points as we require laid down with strict accuracy. The problem which we originally undertook was to transfer to maps all the details supposed to be accurately drawn on a globe 41 feet $9\frac{1}{3}$ inches in diameter. With the meridians and parallels drawn upon this globe the latitude and longitude of every geographical feature can be determined, and if we have a set of skeleton maps showing the meridians and parallels for the whole or any portion of the surface of the globe, we can at once transfer the features from the globe to the maps.

The problem of map projection therefore consists in finding some method of transferring the meridians and parallels from the globe to the map.

This is not a book on surveying or on descriptive geography. We shall therefore not trouble ourselves at all about the actual shapes of the continents and islands or their physical features. We shall confine ourselves to the representation of the meridians and parallels, so that we shall deal only with the simplest geometrical figures, knowing that when these lines have been properly drawn the details of all known lands can be easily filled in. Occasionally, especially in connection with "world maps," we may introduce the land forms in order that the reader may recognise the effect on the familiar forms of continents and countries of the different systems of projection.

The "Terrestrial Globe"

Ordinary terrestrial globes are made hollow and of composition somewhat resembling that used for the heads and shoulders of wax dolls, but instead of being covered with wax they are covered with paper already printed and coloured, so that a coat of varnish is all that is required to complete

the globe when the paper has been cemented to the composition. It was stated above that a sheet of paper cannot be made to fit a globe without stretching, but it was also pointed out that a piece of paper if wetted would stretch sufficiently to fit a small portion of a sphere if pressed against the surface. Most readers have probably at some time made a fire balloon. This is ordinarily done by cutting a dozen gores like that in Fig. 9, and pasting them together along the edges. The widths of the gore in the centre is one-twelfth of the circumference of the required balloon with the addition of a margin for overlap at the pasted joint. The length is one-half the circumference of the balloon. The exact form of the gore, less the margin for pasting, is found by the following process. The length is divided into, say, 18 parts. The breadth of the gore at each of these points of division is one-twelfth of the circumference of the parallel of latitude on a sphere of the size of the balloon corresponding to latitude 0°, 10°, 20°, 30°, 40°, 50°, 60°, 70°, and 80°, as the case may be. At latitude 0°, or the equator of the balloon, the width, as stated above, is one-twelfth of the circumference, if the balloon is to be made of twelve gores. The other breadths can be found at once from a book of trigonometrical tables. They are equal to the central width multiplied by the *cosines* of 10°, 20°, 30°, etc., respectively, and these cosines are given in the tables, but for readers who have no knowledge of trigonometrical ratios, and this book is intended mainly for such, it is sufficient to say that the measurements can be taken along the parallels of latitude of the little ball described on pp. 26–29, and increased in proportion as the diameter of the balloon is to be greater than that of the ball. If the ball were 4 inches in diameter and the diameter of the balloon is to be 4 feet, it will simply be necessary to multiply all the measurements by 12. When these breadths have been set off, half on each side of the central line of the gore, there will be 17 points marked on each curved side of the gore, and it is

Fig. 9.

then easy to draw a smooth curve through each set of 17 points. This curve is known as the curve of sines (see p. 46). In cutting out the gore we shall probably allow a quarter of an inch margin on one side only for the pasted joint, but if the gore were required for covering a solid sphere no lap would be necessary. A simple "butt joint," edge to edge, would be required, and the gore would be cut exactly on the line of the curves as drawn.

The Covering of a "Terrestrial Globe"

In making a fire balloon a circular opening is required at the bottom of the balloon, and therefore at the bottom of each gore the portion between latitude 80° and 90° (the pole) is not required and need not be drawn. The top of the balloon is closed, but if all the twelve gores are brought up to the north pole so that twelve pasted joints meet there, much confusion and thickening of the paper envelope will be produced. It is therefore usual to stop off the gore at or near the parallel of latitude of 70°, and to finish the balloon with a circle of paper cut to the diameter of the 70° parallel of latitude with a margin of, say, a quarter of an inch for pasting. If the paper were required for covering a solid sphere this margin would not be required, but a "polar cap" would be wanted for each pole. When one gore has been cut all the others can be "cut to pattern" as they are all exactly alike, and one pattern can be kept as a templet for making any number of balloons of the same size.

Now suppose a series of gores and two polar caps to be prepared in this way of the exact size to fit a rigid sphere (whether solid or hollow). The fire balloon will not be an exact sphere because each gore of which it is made is "developable" and no part of a spherical surface can be laid out flat, but the greater the number of gores employed, and the narrower the breadth of each, and the smaller the polar caps, the more nearly spherical will the balloon become, just as a regular polygon becomes more and more nearly a circle as the number of sides is increased. Twelve gores, however,

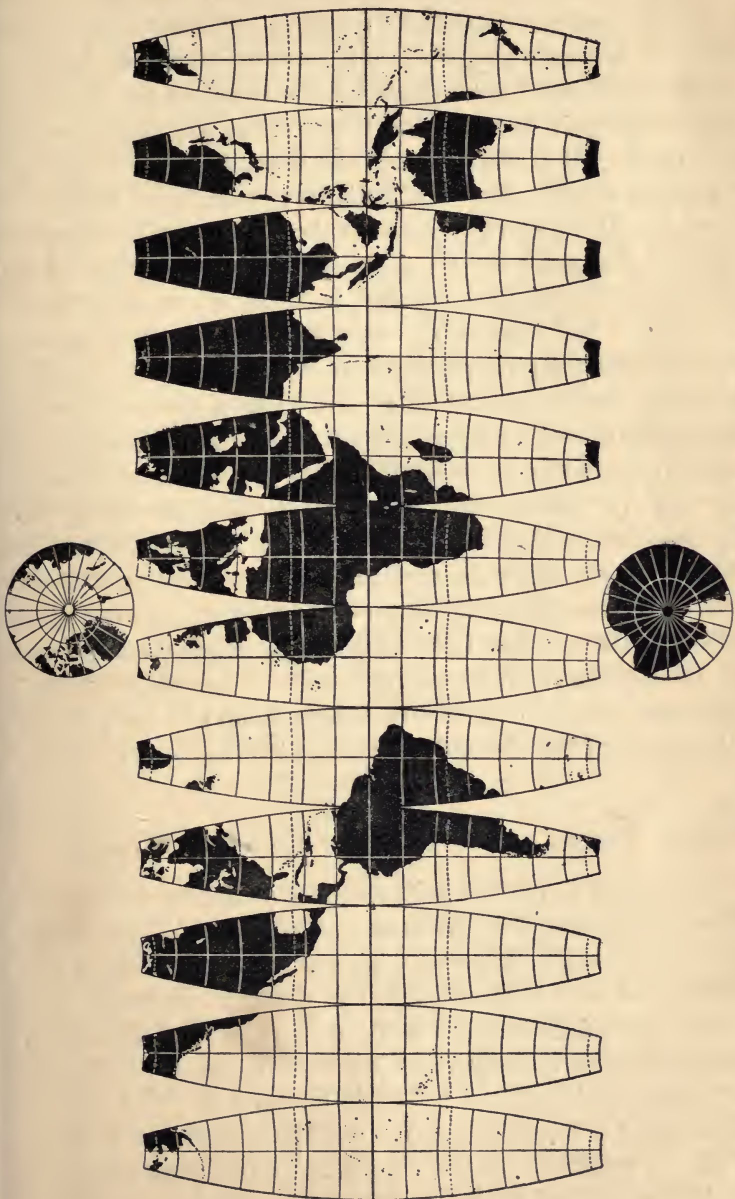


FIG. 10.—THE COVERING OF A "TERRESTRIAL GLOBE."

give a sufficiently accurate sphere for a fire balloon. When the gores and polar caps are used for covering a rigid globe they are wetted, and they will then stretch sufficiently to adapt themselves to the spherical curvature of the globe over their comparatively small width when they are pressed against the surface. It is obvious that the central meridian of a gore is a little shorter than the boundary meridians, whereas on the globe all the meridians are of the same length. Hence, in adapting the gores to the globe the central meridian of each gore must be slightly stretched in comparison with the side meridians. The figure shows on a very small scale the series of gores and polar caps as printed for covering a globe. As printed the gores would exactly fit a globe two inches in diameter. These gores do not constitute a map. They are, as nearly as may be, on a plane surface, a facsimile of the surface of the globe, and only require bending with a little stretching in certain directions, or contraction in others, or both, to adapt themselves precisely to the spherical surface. If the reader examines the parts of the continent of Asia as shown on the separate gores, which are almost a facsimile of the same portions on the globe, and tries to piece them together without bending them over the curved surface of a sphere, the problem of map projection will probably present itself to him in a new light.

The Globe to the Scale of One in a Million

Let us now go back to our system of meridians and parallels. We learned how to draw them on a small ball. We can suppose them drawn in the same way on our large globe which has a diameter of one-millionth of the average diameter of the earth. The diameter of this globe is 41 feet $9\frac{1}{3}$ inches. The distance between parallels of latitude drawn for intervals of 10° on this globe will be 3 feet $7\frac{3}{4}$ inches, and the distance on the equator between successive meridians drawn for intervals of 10° of longitude will be the same. If the meridians and parallels are drawn for every degree the corresponding distance is $4\frac{3}{8}$ inches, so that on a sphere of this size it would not be unreasonable to draw the lines for every degree or even less.

We have introduced and described at some length the globe representing the earth on the scale of one to a million in order that the reader may have before him a clear mental image of a globe on a scale corresponding to the International map, a scale on which the map of Great Britain and Ireland, excluding the Shetland Isles, would be comprised in a sheet measuring 42 inches by 34 inches, but for the study of map projections our equivalent globe may be of any diameter.

A Wire Skeleton Globe

Now suppose the globe to be replaced by a wire cage in which all the meridians and parallels of latitude for every degree are circles of wire. With two circular disks of tinplate for polar caps and steel wire, the reader who has access to soldering material will find it an interesting problem to make a wire globe of this kind about a foot in diameter with wire circles for every 10°. A miniature glow lamp requiring only two or four volts may be held inside the globe in a dark room, and the shadow projections thrown on a screen as described at the beginning of this little book. The essential features of the gnomonic and stereographic projections, and even the cylindrical projection, can then be observed on the shadow pictures. For the cylindrical projection it is necessary to hang the screen as a curtain on a circular rod, and to place the centre of the cage on the axis of the cylinder formed by the curtain. The diameter of the cylinder may be greater than that of the globe, but it is not desirable to make it more than two feet. If this curtain is sufficiently transparent for the shadow to be seen from the outside the whole cylinder may be employed, but otherwise it is not convenient to use more than half the cylinder corresponding to a semicircular curtain rod, and even a smaller portion of the cylinder is sufficient to exhibit all the features of the projection. A cone of tracing paper slipped over the wire cage will enable the conical shadow projection to be examined, and the experiments may be varied in an interesting manner by moving the cone so that its axis is no longer coincident with the polar axis of the cage, but is

inclined to the vertical. An oblique projection will then be obtained.

Definition of Map Projection

It has been stated above that very few of the strictly geometrical or shadow projections are actually used in map-making, and in the term "map projection" the word "projection" is used in a very extended sense. In fact all sorts of liberties are taken with the methods of drawing the meridians and parallels in order to secure maps which best fulfil certain required conditions, provided always that the methods of drawing the meridians and parallels follow some law or system. Hence a map projection may be defined as

A systematic drawing of meridians and parallels for the whole earth or some portion of it, on a plane surface.

Distortion

In order to decide on the *system of projection* we must consider the purpose for which the map is to be used and the consequent conditions which it is most important for the map to fulfil. In geometry, size and shape are the two fundamental considerations. If we want to show without exaggeration the extent of the British dominions on a world map, we do not much mind about the *shape* of Australia, so long as its *area* is properly represented to scale. For statistical purposes, therefore, a map on which all areas are correctly represented to scale is valuable, and such a map is called an "equal area projection." The reader probably knows that parallelograms on the same base and between the same parallels, that is of the same height, have equal areas, though one may be rectangular or upright, and the other very oblique. The sloping sides of the oblique parallelogram may be very much longer than the upright sides of the other, but the areas of the figures will be the same though the shapes are so very different. The process by which the oblique parallelogram can be formed from the rectangular parallelogram is called by engineers "shearing," because it is something like the process by which one piece of metal slides over the other when cut by "shears." A pack

of cards as usually placed together shows as profile a rectangular parallelogram. If a book be stood up against the ends of the cards, as in Fig. 11A, and then made to slope as in Fig. 11B, each

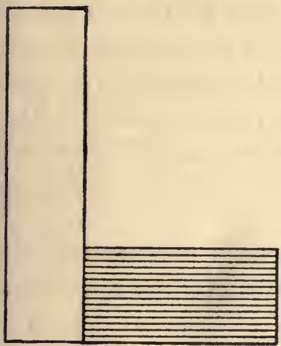


FIG. 11A.

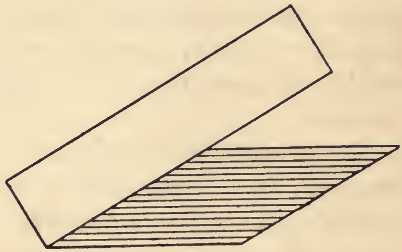


FIG. 11B.

card will slide a little over the one below and the profile of the pack will be the oblique parallelogram shown at B. The height of the parallelogram will be the same, for it is the thickness of the pack. The base will remain unchanged, for it is the long edge of the bottom card. The area will be unchanged, for it is the sum of the areas of the cut edges of the 52 cards. The shape of the parallelogram is very different. The sloping sides, it is true, are not straight lines, but are made up of 52 little steps, but if instead of cards several hundred very thin sheets of paper or metal had been employed, the steps would be invisible and the sloping edges would appear to be straight lines. This sliding of layer upon layer is a "simple shear." It alters the shape without altering the area of the figure.

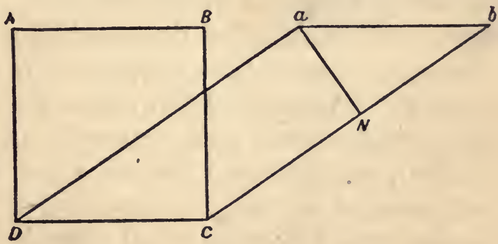


FIG. 12.

This shearing action is worthy of a little more study in order that we may understand one very important point in map projection. Suppose the square, ABCD, to be sheared into the oblique parallelogram, *ab*CD. Its base and height

remain the same and its area is unchanged. But the parallelogram *ab*CD may be turned round so that C*b* is horizontal, and then the C*b* is the base, and the line *a*N drawn from A perpendicular to *b*C is the height. Then the area is the product of *b*C and *a*N, and this is equal to the area of the original square, and is constant whatever the angle of the parallelogram and the extent to which the side BC has been stretched. The perpendicular *a*N, therefore, varies inversely as the length of the side *b*C, and this is true however much BC is stretched. Hence—

In an equal area projection if distances in one direction are increased, those measured in the direction at right angles are reduced in the corresponding ratio.

In map projection this is expressed by saying that if the scale of the map is increased in one direction, it is diminished in the direction at right angles in the inverse ratio. This is the condition necessary for an " equal area projection."

In maps drawn on an equal area, or homolographic projection, some tracts of country may be " sheared " so that their shape is changed past recognition, but they preserve their area unchanged. In maps covering a very large area, particularly maps of the whole world, this generally happens to a very great extent in parts of the map which are distant from both the horizontal and vertical lines drawn through the centre of the map.

It will be noticed that in the shearing process that has been described every little portion of the rectangle is sheared just like the whole rectangle. It is stretched parallel to *b*C, and contracted at right angles to this direction. Hence when, in an equal area projection, the shape of a tract of country is changed, it follows that the shape of every square mile, and indeed of every square inch of this country will be changed, and this may involve a considerable inconvenience in the use of the map. In the case of the pack of cards the shearing was the same at all points. In the case of equal-area projections the extent of the shearing or distortion varies with the position on the map, and is zero at the centre. It usually increases along the diagonal lines of the map. It

may, however, be important for the purpose for which the map is required that small areas should retain their shape even at the cost of the areas being increased or diminished, so that different scales have to be used at different parts of the map. The projections on which this condition is secured are called "*orthomorphic*" (or right-shaped) projections. If it were possible to secure equality of area (uniformity of scale) and exactitude of shape at all points of the map, the whole map would be an exact counterpart of the corresponding area on the globe, and could be made to fit the globe at all points by simple bending without any stretching or contraction which would imply alteration of scale. But a plane surface cannot be made so to fit a sphere. It must be stretched in some direction or contracted in others (as in the process of "raising" a dome or cup by hammering sheet metal) to fit the sphere, and this means that the scale must be altered in one direction or the other or both. It is therefore impossible for a map to preserve the same scale in all directions and at all points; in other words—

A map cannot accurately represent both the size and shape of the geographical features at all points of the map.

Conditions fulfilled by Map Projections

If, then, we endeavour to secure that the *shape* of a very small area, a square inch or square mile, is preserved at all points of the map, which means that if the scale of distances North and South is increased the scale of distances East and West must be increased in exactly the same ratio, we must be content to have some parts of the map represented on a greater scale than others. The "orthomorphic projection" therefore necessitates a change of scale at different parts of the map, though the scale is the same in all directions at any one point. Now it is clear that if in a map of North America the north of Canada is drawn on a much larger scale than the Southern States, although the *shape* of every little bay or headland, lake or township is preserved, the shape of the whole continent on the map must be very different from its shape

on the globe. (Compare North America or Asia or simply Greenland on Mercator's map of the world with the same areas on the school globe.) In choosing our system of map projection therefore we must decide whether we want—

(1) *To keep the areas directly comparable all over the map at the expense of the correct shapes* (*equal-area projection*), *or*

(2) *To keep the shapes of the smaller geographical features, capes, bays, lakes, etc., correct at the expense of a changing scale all over the map* (*orthomorphic projection*) *and with the knowledge that large tracts of country will not preserve their shape, or*

(3) *To make a compromise between these two conditions so as to minimise the errors when both shape and area are taken into account.*

There is a fourth consideration which may be of great importance, and is very important to the navigator, while it will be of much greater importance to the aviator when aerial voyages of thousands of miles are undertaken, and that is that the directions of places taken from the centre of the map, and as far as possible when taken from other points of the map, shall be correct. It has been pointed out that the horizontal direction of an object measured from the north is known as its azimuth. Hence a map which preserves these directions correctly is called an "*Azimuthal Projection.*" We may therefore add a fourth object, viz :—

(4) *To preserve the correct directions of all lines drawn from the centre of the map* (*azimuthal projection*).

For a not very satisfactory reason, which we shall discover later on, *azimuthal* projections are commonly called "*zenithal*" projections.

We have now considered the conditions which we should like a map to fulfil, and we have found that they are inconsistent with one another. For some purposes we may construct a map which fulfils one condition and ignores another, or *vice versâ ;* but we shall find that the maps most commonly used, as is the case in most political questions, are the result of compromise, so that no one condition is strictly fulfilled, nor is it extravagantly violated.

WORLD MAPS

Sanson-Flamsteed Sinusoidal Projection

As it is usual in atlases for the first map or first two or three maps to be maps of the whole world, it may be desirable to consider first the systems of projection by which such maps can be drawn. The first method we shall describe is selected because it affords an illustration of a "projection" which differs very widely from any of the geometrical, or shadow, projections which we have described, while its principles are easily understood. Though the method is more than 250 years old, world maps on this principle are not to be found in any atlases, but such maps have recently been published on separate sheets. Wall maps and atlas maps of Africa and South America are very common on this projection and maps of Australia and Polynesia have also been published. These maps can be recognised by the following characteristics:—

> The parallels of latitude are horizontal straight lines drawn at equal distances and each parallel is equally divided by the meridians, which are slightly curved so as to be nearer together the higher the latitude.

The projection is an equal-area projection and it is generally known as the Sanson-Flamsteed Sinusoidal projection, but there is no reason to believe that Flamsteed, the Astronomer Royal, had any part in its invention, although he used it in his star maps. Sometimes it is simply called the sinusoidal projection, because all the meridians are of the form of curves of sines. A curve of sines is the profile of the simplest form of wave. It is the form into which a uniform and straight steel spring will bend if the ends are squeezed together. If a bow were made of uniform section instead of tapering to the ends, the string would bend it into a curve of sines. If a weight is suspended by a steel spring, and made to dance up and down, and a pencil is attached to

it while a sheet of paper is drawn horizontally past the pencil with uniform speed a curve of sines will be drawn on the paper. If a circle of wire is made to spin round at constant speed about a diameter in a uniform magnetic field like that of the earth, there is an alternating current produced in the wire, and if a "graph" be drawn to show the rise and fall of the current with the time as base, the graph will be a curve of sines. The best generators which produce alternating electric current for lighting or power produce a current which varies so that its graph is a curve of sines, and a clock pendulum swings so that its distance from the vertical position at any instant follows

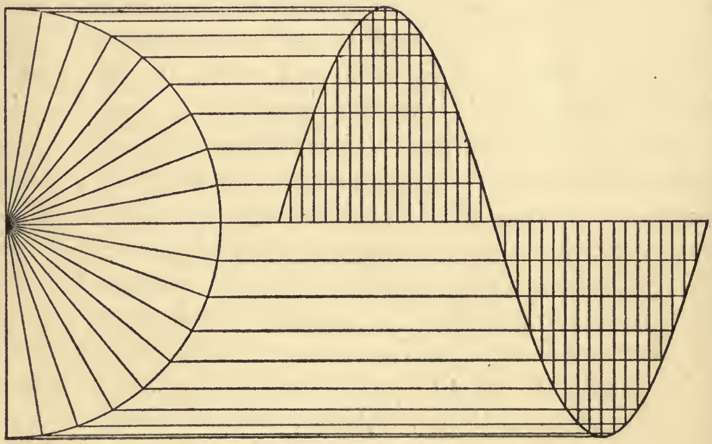


Fig. 13.—A Curve of Sines.

the law of the curve of sines. When a musical string is vibrating so as to produce a pure note its form at any instant is a curve of sines. The curve of sines, therefore, is of such vast importance in physics and mechanics that it is worth studying, apart from its use in map projection.

The curve of sines may be as sharp as we please, or may be very shallow, just as there may be ocean waves the sides of which are very sharp or long rollers the crests of which are many yards apart while the rise and fall is only a few inches. The way to draw a curve of sines is first to draw a circle with radius equal to the greatest height or depth of the required curve above or below the base line. Draw the vertical

diameter and divide the semi-circumference into any number of equal parts, say 20, by means of the protractor. Now draw a horizontal straight line of length equal to the base of the desired curve. This may be as long or as short as you please. Divide the line into the same number of equal parts as the circumference of the circle and draw through these points vertical straight lines. Through the points of division of the semicircle draw horizontal straight lines. They will meet these vertical lines as shown in Fig. 13, in points on the curve of sines. The radius of the circle is called the "*amplitude*" of the curve or wave. The length of the wave is that of the original base line. As the circle may be as large as you please with the same base line the curve may be as steep or as shallow as you please. When water waves become very steep they break up into "breakers."

To draw a sinusoidal projection of the world, first draw the horizontal straight line EQ, representing on the required scale the whole length of the equator. Bisect it at C, and draw NC and CS perpendicular to EQ, and each equal to the length of the meridian from Pole to Equator. Thus NS is half of EQ. Divide NS into 18 equal parts to correspond each to 10° of latitude. Through each of these points of division draw a horizontal straight line, and while each line is bisected by NS make the lines respectively equal in length to the circumferences of the parallels of latitude for 10°, 20°, etc. Through the extremities of these lines draw curves. These curves will be the boundaries of the world map. Divide every parallel, including the equator, into 36 equal parts, and through the corresponding points of division on each successive parallel draw a curve. These curves will be meridians for each 10° of longitude, and the system of meridians and parallels, that is, the map projection, is complete. The set of meridians and parallels drawn according to this or any other system is sometimes called a "graticule." To complete the map it is only necessary to insert the geographical features according to their latitude and longitude. (See Fig. 14, p. 48.)

If you consider any small quadrilateral into which the map is divided you will see that its horizontal length measured

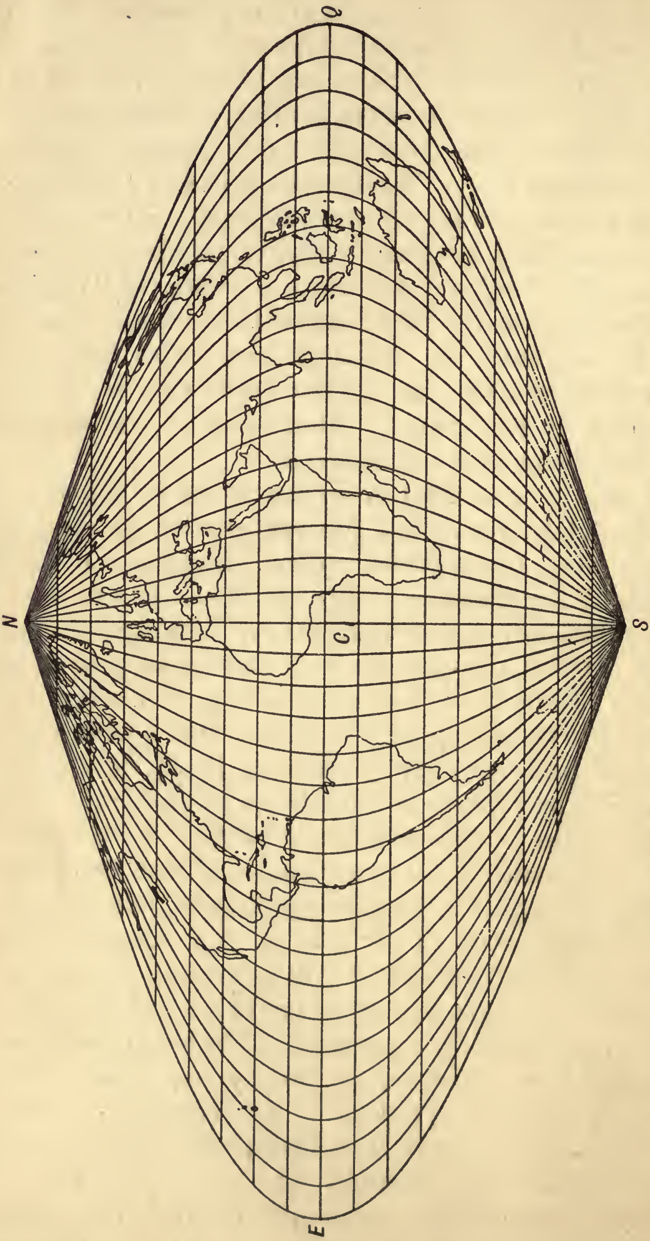


FIG. 14.—SANSON-FLAMSTEED SINUSOIDAL PROJECTION.

along either its top or bottom side is the precise length of the corresponding quadrilateral on the globe measured along its bounding parallels of latitude, while the vertical height of the quadrilateral on the map is the same as the distance measured on the globe along the meridian, and therefore at right angles to the parallels of latitude. Hence the area of the quadrilateral on the map is the same as that of the quadrilateral on the globe between corresponding meridians and parallels of latitude. As this is true of every quadrilateral into which the map and the globe are divided, it follows that the whole area of the map is the same as the whole area of the globe made to the same scale, and every figure drawn on the map has the same area as the figure on the globe, the outline of which corresponds point for point in latitude and longitude. The sinusoidal projection is therefore an equal-area projection, and it is obtained from the globe by the simplest possible process.

It will be noticed that while the quadrilaterals along the central meridian and the equator retain the same shape as on the sphere, as we move farther and farther to the right and left along any parallel of latitude other than the equator, the quadrilaterals become more and more distorted or sheared, and the distortion becomes very great in high latitudes (near the poles) and longitudes remote from the central meridian. It is this excessive distortion which makes the projection of little value for whole world maps. It should further be noticed that all the meridians, like the boundaries of the map, are curves of sines varying only in amplitude. The extreme meridians forming the boundaries of the map meet at the poles at an angle of nearly 145°.

The central meridian of the graticule may be taken to be the Meridian of Greenwich or any other as we please. If we wish the map to show a particular tract of land with the least possible change of shape, we may take the meridian through the middle of that country as central meridian, and the distortion of that country will then be very slight provided that the country does not extend very far east and west, or lies very close to the equator. Thus we can get a very good map of Africa or South America by placing the middle of the

continent on the central meridian, for over the area covered by these continents, the greatest breadth of which lies very close to the Equator, the quadrilaterals are very little distorted. This is the reason why the sinusoidal projection is so often chosen for maps of these continents. Australia is too far from the equator, and extends too much east and west to allow of a good map on this system, while Asia is quite unsuitable for the sinusoidal projection. Figs. 14 and 15 show maps of the whole world and of Africa on the Sanson-Flamsteed sinusoidal projection. This projection may be employed to show the world in two hemispheres without very great distortion.

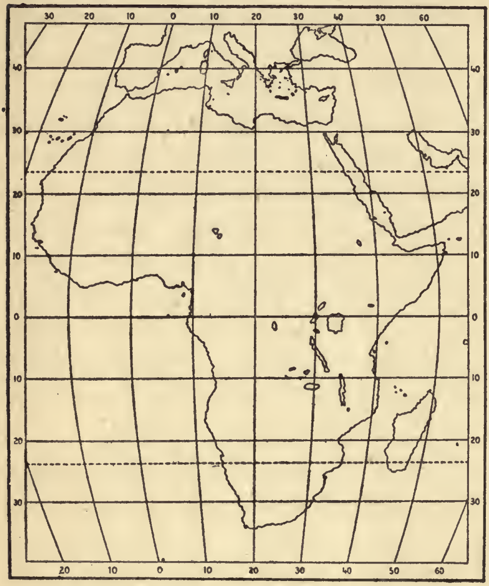


FIG. 15.—AFRICA ON THE SINUSOIDAL PROJECTION.

MOLLWEIDE'S EQUAL-AREA PROJECTION

The next system we propose to describe is still more removed from a geometrical projection, and is much more difficult to understand and to draw, but it will be found in many school atlases. It is Mollweide's equal-area projection. Unlike most other maps, it is commonly provided with its full title in the atlases, but you can recognise it because its boundary is an ellipse, and the parallels of latitude are horizontal straight lines, though they are not spaced at equal distances. All the meridians other than the central meridian are ellipses except one, which is a circle.

It is proved in books on mensuration that the area of the

surface of a sphere is four times that of a great circle, and therefore the area of a hemisphere is twice that of its circular base. This indicates the amount of "stretching" required to raise a circular plate of metal so as to form a hemispherical bowl. It has to be hammered till its thickness is reduced to one half, and its area doubled without stretching the rim. The area of a circle is πR^2, where R represents the radius and π is 3·1416 nearly, or approximately $3\frac{1}{7}$. Now suppose that we have a "terrestrial globe" constructed on the scale to be adopted for our map, and let its radius be R inches. Draw a circle of radius $\sqrt{2}R$ inches, or 1·4142R inches. The area of this circle will be $\pi(\sqrt{2}R)^2$, or $2\pi R^2$, square inches, which is the area of one half of the sphere. Let C be the centre of the circle, and draw the horizontal diameter ACB, and the vertical

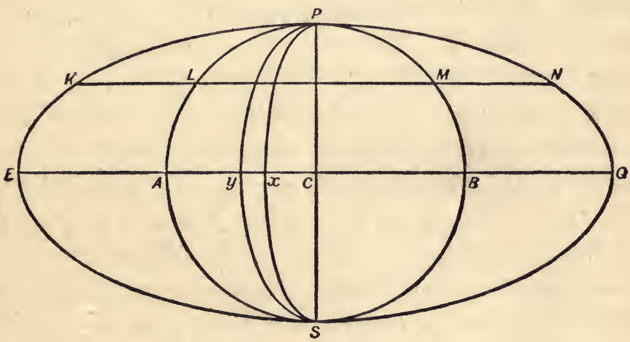


FIG. 16.

diameter PCS. Produce AB both ways so that EC is double of AC and QC of BC. Draw an ellipse, having EQ and PS for axes. Then it is one of the properties of the ellipse that if any line be drawn parallel to EQ cutting the ellipse and circle in K, N, and L, M respectively, KN is double of LM, and as this is true for every horizontal line, or narrow strip, it follows that the area of the ellipse is double that of the circle, and is therefore *equal to the area of the whole globe.* In this figure PS will represent the central meridian from pole to pole, and EQ the equator. Now divide EQ into thirty-six equal parts. The points ABC will be among the points of division. Through the other points of division with PS as an axis draw ellipses.

These will be the meridians. From the properties of the ellipse it follows that all these curves will cut any horizontal straight line like KLMN at equal intervals, and hence all the gores, such as $PxSy$, will be of equal area. Consequently the area of each of these gores will be one thirty-sixth of the whole area of the map, that is, of the globe, and will be equal to the area on the globe comprised between consecutive meridians drawn at intervals of 10°. For an equal-area projection it remains to divide these gores by parallels of latitude into the same areas as the corresponding gores between the meridians are divided on the globe. This is the difficult part of the problem, and it requires a little trigonometry to obtain a neat solution. Our object, however, is not to solve mathematical problems, but to describe map projections so that they can be generally understood, and their special properties recognised.

Consider again the sinusoidal projection. Let it represent a sphere of radius R. The area of the whole map is that of the sphere, $4\pi R^2$. The length of the equator, EQ, is $2\pi R$ or $6{\cdot}2832R$ and the length of the meridian, PS, is πR or $3{\cdot}1416R$. In the Mollweide projection the major axis of the ellipse was not made equal to the length of the equator, but to $4\sqrt{2}R$ or $5{\cdot}6568R$, and the length of the central meridian was made one half of this or $2{\cdot}8284R$. Hence if the Mollweide ellipse be superposed on the corresponding sinusoidal projection, and be represented by $pqse$ the central meridian, ps is short of PS by $(3{\cdot}1416 - 2{\cdot}8284)R$ or $0{\cdot}3132R$, and therefore Pp or Ss is equal to $0{\cdot}1566R$. The intercepts, Ee and Qq, are each double of this, or $0{\cdot}3132R$. But as the areas of the two figures are equal, being each equal to the area of the sphere they represent, it is clear that the ellipse cannot lie entirely within the boundary of the sinusoidal projection, and it must overlap in the way shown in the figure. If parallels of latitude are drawn at equal distances on the sinusoidal projection, we know that the areas of the strips between them are equal to the areas of the corresponding zones on the globe, but if the semi-axis ps of the ellipse be divided into eighteen equal parts, and parallels be drawn through the points of the division, it is clear that the parallels in the ellipse near the equator will be shorter than

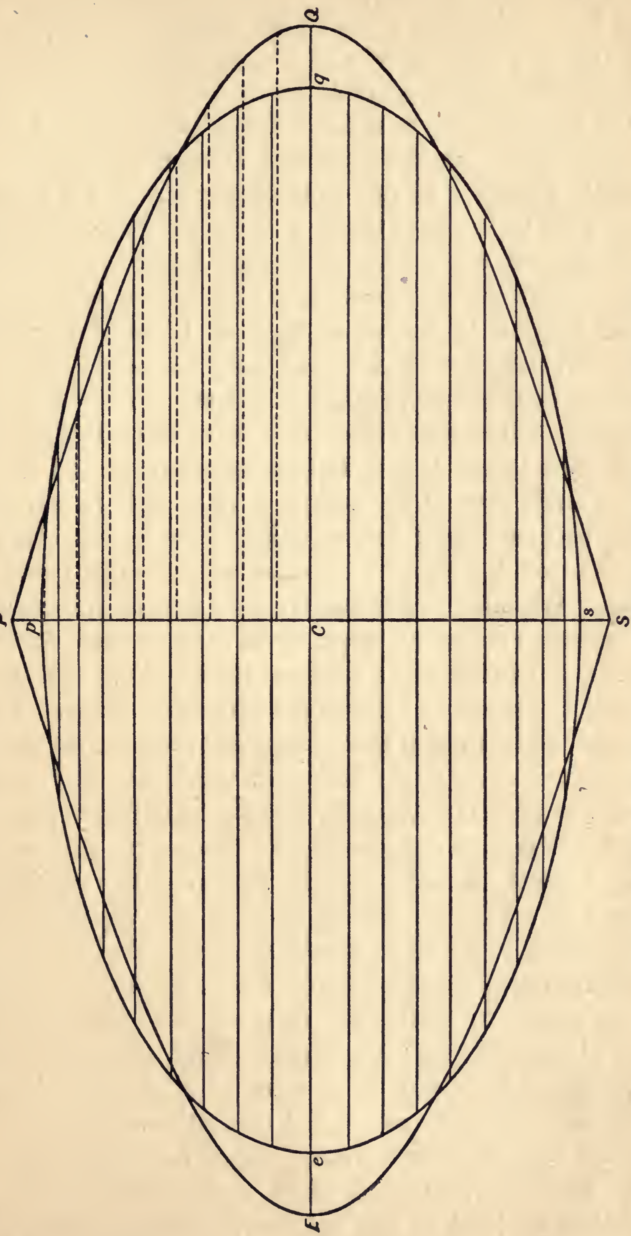


FIG. 17.—MOLLWEIDE'S AND SINUSOIDAL PROJECTIONS SUPERPOSED.

those of the sinusoidal projection, while the distance between them is less. The first strip of the ellipse, between 0° and 10° latitude (or between 0° and 1° latitude) will have an area less than that of the sinusoidal map on both accounts. But we want the two areas to be equal. Therefore we must make the breadth of the strip in the ellipse greater than that in the sinusoidal projection in the same proportion as EQ is greater than *eq*, that is, in the ratio of π to $2\sqrt{2}$. Proceeding in this way for very small intervals of latitude instead of intervals of 10°, we may divide the ellipse into very narrow strips each of the same area as the corresponding strip in the sinusoidal projection, and therefore having the same area as the zone of the globe between the same parallels of latitude. At first the strips in the ellipse will be wider than those of the sinusoidal projection because their length is less, but as we approach the pole the ellipse extends beyond the curves of sines and the parallels of latitude become longer in the ellipse than in the other projection, so that the widths of the successive strips become less and less. We must finish exactly at the pole, p, because we know that the area of the ellipse is the same as that of the other projection. The distances between the parallels of latitude in Mollweide's projection therefore decreases as the latitude increases. This means that even on the central meridian small tracts of country are stretched vertically and contracted horizontally near the equator, and stretched horizontally and contracted vertically near the poles. There are only two points on the central meridian where the map is orthomorphic. Moreover, at all points, except those on the central meridian and the equator, the little quadrilaterals formed by the meridians and parallels are distorted so as to be no longer rectangular. Fig. 17 shows a Mollweide map of the world with the Sanson-Flamsteed projection, drawn to the same scale, superposed. Mollweide obtained a formula for finding the proper position of each parallel of latitude on the vertical axis of the ellipse by calculation from Trigonometrical Tables. In Fig. 17 the full horizontal lines represent the parallels in Mollweide's projection, and the broken lines in one quadrant represent the equidistant parallels in the sinusoidal projection,

and what has been said about the variation of scale will be apparent from the positions of the parallels.

These projections and nearly all the other projections in this book are drawn so as to correspond to a globe two inches in diameter, so that R is one inch, and all these projections are, therefore, directly comparable with one another. The diameter of this globe is almost exactly 1/250,000,000 of that of the earth. The length of the equator is 6″·283 on the two-inch globe. Hence in the Sanson-Flamsteed projection the length of the map is 6″·283, and the height 3″·142, but in the Mollweide's map the equator is reduced to 5″·66, and the distance from pole to pole is 2″·83.

The reader who has a very elementary knowledge of Trigonometry will be able to understand the following method of procedure, but this paragraph may be omitted with impunity.

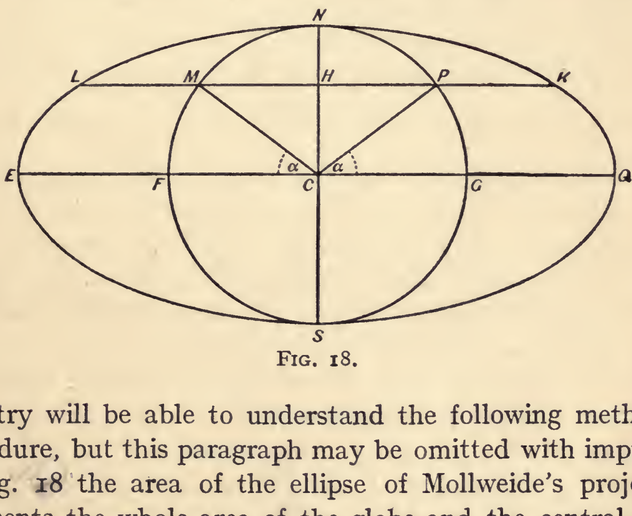


FIG. 18.

In Fig. 18 the area of the ellipse of Mollweide's projection represents the whole area of the globe and the central circle represents the area of the hemisphere. Moreover, if any horizontal line LMPK be drawn LK is double of MP, and therefore the area LKQE is double of the area MPGF. We want to draw LK so that it may correspond to some particular degree of latitude, ϕ on the globe. Let Fig. 19 represent the globe. Make the angle BCQ equal to ϕ and draw the parallel of latitude, AB. Then the

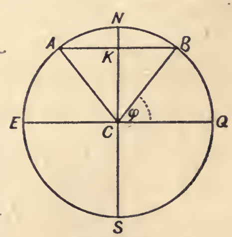


FIG. 19.

area of the zone between the equator and AB is equal to the circumference of the equator, $2\pi R$, multiplied by KC (see p. 59), that is $2\pi R^2 \sin \phi$. We want the plane area MPGF in Fig. 18 to be one half of this. But the radius CG is $\sqrt{2}R$. Call this r. Then the area MPGF is made up of the two equal sectors PCG and MCF, and the triangle CMP. The area of each sector is $\frac{1}{2}r^2\alpha$ if α be measured in radians, or $\frac{1}{2}r.PG$, and the area of the triangle is $\frac{1}{2}.MP.HC$, that is, $r^2 \cos \alpha \sin \alpha$. Hence the whole area MPGF is $r^2\alpha + r^2 \cos \alpha \sin \alpha$. But $r = \sqrt{2}R$, therefore $r^2 = 2R^2$. Hence the area is

$$2R^2(\alpha + \cos \alpha \sin \alpha) \text{ or } R^2(2\alpha + \sin 2\alpha).$$

This has to be equal to $\pi R^2 \sin \phi$, one half the area of the zone on the sphere. Therefore the angle α is to be determined from the equation $2\alpha + \sin 2\alpha = \pi \sin \phi$. It is not easy to obtain α in terms of ϕ from this equation, but it is quite easy to determine ϕ in terms of α, for $\sin \phi = \dfrac{2\alpha + \sin 2\alpha}{\pi}$. Hence if any parallel be drawn in Mollweide's ellipse and the angle α measured, the latitude, ϕ, to which it corresponds, is found at once. If all the areas corresponding to LKQE in the ellipse are equal to the areas of the corresponding zones on the sphere, then the area between any two parallels on the ellipse will be equal to the area of the corresponding zone on the sphere, and since the meridians divide all the parallels in the ellipse equally we have succeeded in obtaining an equal-area projection.

Suppose a set of gores to be prepared for covering a globe as described on p. 36, except that the whole surface is divided into a great many more than twelve gores, and each gore is continued to the polar caps. Then if the gores are placed so that the equator is a straight line, and are all divided into very thin strips parallel to the equator, and these strips are made to " right and left close " upon the centre meridian until the strips are in contact so that there is no gap in the picture, the result will be the Sanson-Flamsteed map of the world. If the strips are narrow enough, and there are a large number of gores, the little steps at the points of contact of

adjoining strips will be invisible. The sinusoidal projection can, therefore, be formed by simply shearing the gores. Those which are farthest from the central meridian are most sheared, but in each gore there is no shearing close to the equator. It increases towards the poles where the space covered by the "right and left closing" is greatest. The writer has actually made a rough sinusoidal map in this way from the gores provided for the purpose of covering a globe. But while the sinusoidal projection can be obtained by "simple shearing" from the globe gores this is not the case with the Mollweide projection. In addition to the shearing the strips have generally to be expanded vertically and contracted horizontally or contracted vertically and expanded horizontally according to their distance from the equator. Mollweide's projection may also be used for hemispheres or smaller areas.

The Globular Projection

The world in hemispheres is to be found in every atlas. It is often drawn on the so-called "globular projection." This is a very uninteresting projection, and has nothing to recommend it except the simplicity of its construction. A circle is drawn to represent each hemisphere. In each circle the horizontal diameter represents one half the equator and the vertical diameter the central meridian. Divide each equatorial diameter into eighteen equal parts to represent intervals of 10° of longitude. Divide each central meridian similarly into eighteen equal parts to represent intervals of 10° of latitude. A circle can be drawn through any three points. Draw a series of circular arcs through the poles, and each point of division of the equatorial diameters, except the centres, where the circles become straight lines. These are the meridians. Divide each quadrant of the boundary circles into nine equal arcs. Through pairs of these points equidistant from the poles and the corresponding points of division of the central meridians draw circular arcs. These represent the parallels of latitude. The "graticule" is now complete, and the land masses may be drawn in. The system has no feature of interest except that it is adopted in

atlases, and is common in wall maps, so that it is unnecessary to illustrate it here.

It will be noticed that if the boundary circle of the map of a hemisphere on the globular projection have the same diameter as the globe to which it corresponds the area of the map will be only one half that of the hemisphere. If the area of the whole map is to be the same as that of the hemisphere of a globe of radius R, then the radius of the circle of the hemisphere must be $\sqrt{2}R$. The length of a degree of latitude or longitude at the centre of the map will then be $\frac{2\sqrt{2}R}{180}$ or $\frac{\sqrt{2}R}{90}$ and the area corresponding to one degree of latitude and longitude will be $\frac{2R^2}{8100}$. But on the globe the length of a degree of latitude or longitude at the equator is $\frac{\pi R}{180}$, and the corresponding area is $\frac{\pi^2 R^2}{180^2}$. As π^2 is very nearly equal to ten, this is nearly equal to $\frac{2{\cdot}5R^2}{8100}$. Hence the area at the centre of the map is only four-fifths of the corresponding area in the globe, and near the periphery the scale of areas must be increased since the area of the whole map is equal to that of the hemisphere. In the case of the map with a diameter the same as that of the globe, an area near the centre is only two-fifths of the corresponding area of the sphere.

Cylindrical Equal-Area Projection

The cylindrical equal-area projection has many points of interest connected with the geometry of the sphere, but the distortion involved in high latitudes is so great that it is of no value as a world map. We will suppose that we have a hollow glass sphere on which the meridians and parallels together with the land areas and other principal features are painted in transparent colour, or we may content ourselves with the wire skeleton globe, giving only the meridians and parallels. Let the screen be bent into a cylinder touching the globe all round the equator. Now suppose that we have a tiny electric light

surrounded by a lens which causes all the light to be projected in a horizontal plane. (Lighthouse lenses are generally made so that the light emitted deviates from the horizontal plane by a very small angle only.) Let this light be placed in the polar axis of the sphere and moved from the north pole to the south. In any position it will throw upon the screen a narrow horizontal circular band of light intersected by the shadows of the details on the globe which come within the thin disk of light. As the light moves let these details be drawn on the screen. When the screen is opened out we shall have a world map which is a cylindrical equal-area projection.

We may understand the method of projection equally well by dispensing with the light and the lens, and by supposing straight lines drawn horizontally through the polar axis and every point on the globe which we wish to represent on the map, and these lines produced to meet the cylinder at points which will be the corresponding points on the projection. It is clear that the cylindrical equal-area projection is a true geometrical projection.

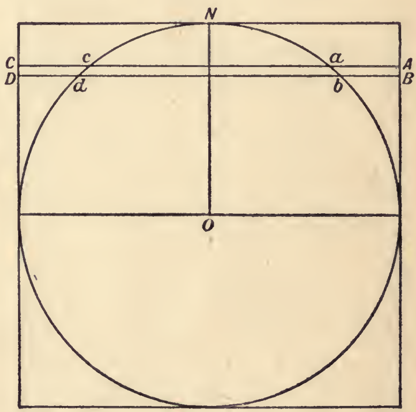


FIG. 20.

Suppose a sphere to be placed in a cylindrical box (a pill box) which just fits it, and touches it all round the equator. Suppose both sphere and cylinder to be cut by two parallel planes, C*ca*A and D*db*B, which are very near together and horizontal. A very narrow zone *cabd* will be cut from the surface of the sphere, and another narrow ring CABD from that of the cylinder. On the cylindrical equal-area projection just described the ring on the cylinder will be the projection of the zone on the sphere. Now the circumference of the cylinder is equal to the equator of the sphere and greater than the circumference of the small circle *db*. On the other hand, the breadth, AB, of the ring on the cylinder is less than the breadth,

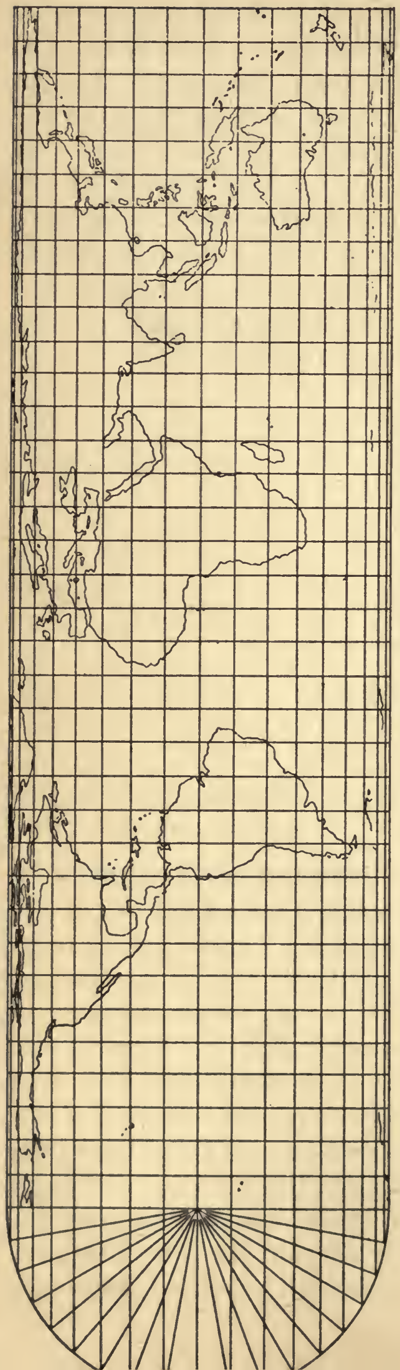


FIG. 21.—CYLINDRICAL EQUAL-AREA PROJECTION.

ab, of the zone on the sphere, for AB is perpendicular to the two planes, and *ab* is oblique. It is easy to prove geometrically that when the distance between the planes is very small, AB is less than *ab* in exactly the same ratio in which *db* is less than DB or the product of AB, and the circumference of the cylinder is equal to the product of *ab* and the circumference of the zone. Hence the area of the cylindrical ring is equal to the area of the spherical zone. This will be true of every narrow belt into which the sphere and cylinder may be divided by horizontal planes. It is, therefore, true of the sum of all the spherical zones and cylindrical rings; in other words, the area of the sphere is equal to the curved area of the cylinder. But if R be the radius of the sphere the height of the cylinder is 2R, and its circumference is $2\pi R$. Hence the area of the cylinder is $4\pi R^2$, which is, therefore, the area of the sphere. This is the easiest way of proving that the

area of a sphere is four times the area of its equatorial circle.

But not only is the whole area of the cylinder equal to the whole area of the sphere, but the area of each ring of the cylinder is equal to the area of the corresponding zone of the sphere. It has been pointed out that the ring of the cylinder is the projection of the corresponding zone of the sphere on the system just described. Hence, on this projection the area between any two parallels of latitude is equal to the area on the sphere between the same parallels, and it follows that the projection on the cylinder of any figure on the sphere will have the same area as the figure. It will be noticed, however, that all parallels of latitude on the sphere, however small, are represented on the cylinder by circles of the size of the equator, and the pole itself is represented by the whole rim of the cylinder. Hence on this projection, though the equality of areas is maintained throughout, there is excessive horizontal stretching and vertical compression in high latitudes, so that the projection is very far from orthomorphic except near the equator. The map, Fig. 21, will be contained within a rectangle $2\pi R$ in length and $2R$ in height. The meridians will be vertical straight lines at equal distances. The parallels of latitude will be horizontal straight lines which will be nearer together the higher the latitude.

Mercator's Projection

Most atlases contain a World Map on Mercator's projection. The maps as printed are only part of the World Map, for on this system the poles are projected to infinity and latitudes above $83\frac{1}{2}°$ are not usually represented. This is not of much importance as land above this latitude is of little value, and few persons desire to travel there. In Mercator's projection countries in high latitudes are shown on a much larger scale than those near the equator, and Canada compares very favourably as regards area with the United States. Mercator's is therefore a favourite projection for British Empire maps. It has two other merits. It is orthomorphic, though large areas

do not appear of their right shape, and a ship's course which is directed so as always to be the same by compass, that is, always to make the same angle with the meridians, is shown by a straight line, sometimes called a rhumb-line, and sometimes called a loxodrome (sloping course), on the map. Rhumb is an old word for a compass point, and a rhumb-line course is a course directed always to the same point of the compass.

Mercator's projection belongs to the cylindrical type, all the meridians being parallel and equidistant straight lines. In connection with the cylindrical equal-area projection it was pointed out that the parallels of latitude are magnified more and more as they approach the poles, for they are all of the same length in that projection. In the equal-area projection, however, it was found that the distances between the parallels was reduced in projection as the length of the parallels was increased and this gave equal areas with great distortion in high latitudes. In Mercator's projection exactly the opposite course is adopted. The distances between the parallels are increased in the same ratio as the parallels themselves are magnified. Thus the vertical scale is increased in exactly the same way as the horizontal scale. In latitude 45° each scale is increased, as compared with the scale at the equator, in the ratio of 1 to $\sqrt{2}$, and hence all small areas in latitude 45° are doubled on the map, for both length and breadth is increased $\sqrt{2}$ times. In latitude 60° each scale is doubled, and areas are therefore increased fourfold. In latitude 75° each scale is increased nearly four times, and the areas nearly sixteen times, and in latitude 80° areas are increased thirty-three times.

Suppose NAB to represent one half of a gore of the form, used for covering a terrestrial globe so that NA and NP are meridians, and let them be 10° apart, so that AB is one thirty-sixth of the equatorial circumference. Let NM be the central meridian. Divide NM into nine equal parts, and through the points of division draw the horizontal parallels. These will correspond to each 10° of latitude. Now suppose that the half gore is made of malleable metal and that geographical features are stained on the surface in such a way that they

are not destroyed by hammering. Suppose that the plate is so hammered that at every point it expands equally in length and breadth as its thickness is reduced, and let the amount of hammering be so regulated that throughout its whole length, as far as the hammering is continued, the breadth of the gore is increased so as to be equal to AB. It is clear that this process cannot be carried out to the point N, for as the width of the gore becomes indefinitely small at the vertex it would have to be hammered infinitely thin to make it expand to the breadth AB; it will therefore be well to cut off the top of the gore at, say, the parallel of 80°. Now under the hammer every little strip parallel to AB expands equally in that direction and in the direction of NM. Hence the metal plate lengthens as its several portions are made broader. For example, at 45° the length of the strip is increased in the ratio in which AB is greater than the breadth of the gore at the parallel of 45°, that is as $\sqrt{2}$ is greater than 1. Hence both the length and breadth are increased $\sqrt{2}$ times, and therefore the area of the strip is doubled, and its thickness halved. A narrow strip on the parallel of 60° will be doubled in length, and therefore doubled in breadth, and its area multiplied four times, while its thickness is reduced to one-fourth. Generally, every little horizontal strip is increased both in length and breadth in proportion as the radius of the sphere is greater than the radius of the circle of latitude upon the sphere on which the strip is situated, and its area is, therefore, increased and its thickness diminished in the square of that ratio. At 30° the area is increased to four-thirds and the thickness reduced to three-quarters. At 80° the area is increased and the thickness reduced about 33·1 times, and at 85°, about 131·6 times. The figure shows the result of the hammering. The metal plate between the equator and latitude 80° is hammered

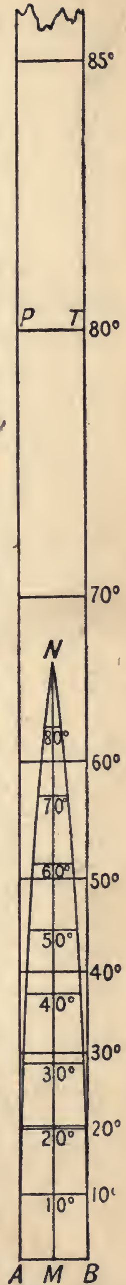


FIG. 22.

out into the rectangle PTBA, and while the thickness at the equator remains unaffected, at PT it is reduced to less than one thirty-third, say 3 per cent., of its original value. If all the gores shown in Fig. 10, p. 37, were extended to latitude 80°, instead of latitude 70°, both north and south and were placed side by side as in that figure, and subjected to this treatment, they would unite to form a rectangle the height of which would be twice PA, and the length the equatorial circumference of the globe they were designed to cover. If the map drawn on the metal gores survived the treatment, it would form a Mercator Map of the World between the parallels of 80° N. and 80° S. If any very small figure, a square or a circle, were drawn on one of the gores, since it would expand equally in all directions, it would remain a square or a circle. Hence Mercator's projection is orthomorphic, or very small areas retain their shape. If the figure extended through a sensible interval of latitude, it would be made to expand more in the higher latitude than in the lower, and therefore would not retain its shape, so that large areas do not retain their true shape on Mercator's projection. In fact, we have seen that while at the equator the areas are unchanged, in latitude 80° they are increased more than thirty-three times. Fig. 22 shows the parallels on the Mercator projection corresponding to each 10° of latitude, and, therefore, to the parallels on the gore, with the addition of the parallel of 85°.

The distance of each parallel from the equator can be exactly calculated from the condition of equal extension at each point along and perpendicular to NM. In the note on p. 101 this problem is worked out in terms of the notation of the integral calculus, and an expression for the distance of each parallel from the equator for intervals of 10° is found. The distances of the parallels from the equator in terms of the radius are approximately for—

10°	20°	30°	40°	50°	60°
0·1755R	0·3563R	0·5493R	0·7632R	1·0112R	1·3171R

70°	80°	85°	90°
1·7358R	2·4366R	3·1316R	∞

making no allowance for the ellipticity of the Earth.

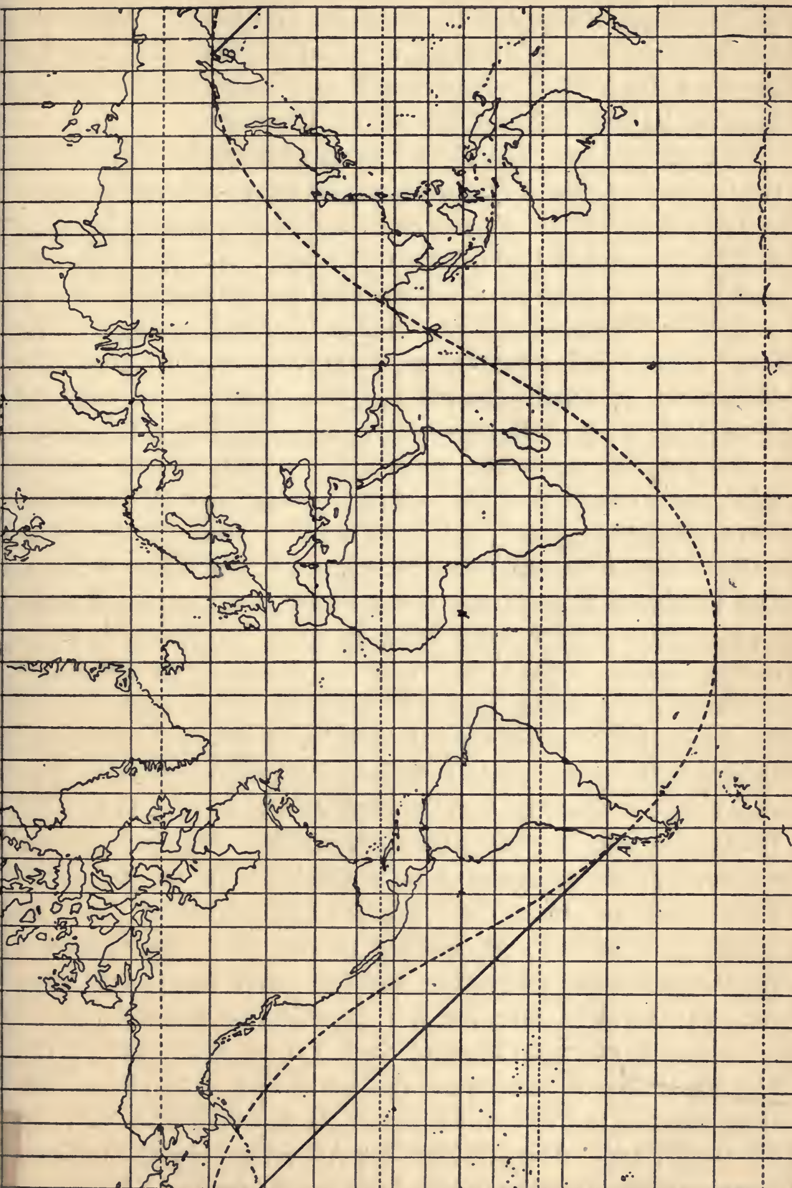


FIG. 23.—MERCATOR'S PROJECTION.

In Mercator's projection the meridians are equidistant vertical straight lines. The parallels of latitude are horizontal straight lines, but the distances between them increase more and more rapidly as they recede from the equator, and the distance between latitude 89° and latitude 90° is infinite.

As stated above the special merit of Mercator's projection lies in the fact that a uniform compass course is represented by a straight line, so that a mariner on an ocean voyage has only to draw a straight line between two ports, and the angle this makes with the meridian, on the map (or chart) gives his true course for the whole voyage. But the uniform compass course is not a great circle except along the equator or along a meridian (due north or south), and it has been pointed out that the shortest distance on the surface of a sphere between two points is the shorter arc of the great circle passing through them. With the exception of the equator, which cuts all meridians at right-angles, a great circle does not cut successive meridians at the same angle, and in a long voyage, in which both latitude and longitude may change considerably, the compass bearing on the great circle is very different at the beginning and end of the voyage. The great cost of fuel and establishment expenses on steamers makes it important that every means should be taken to shorten the length of ocean voyages, and therefore the great circle is followed whenever possible, provided that it does not involve danger from icebergs, as in the North Atlantic, or any other difficulties. The great circle, however, is not represented by a straight line on Mercator's projection, and great circle courses are calculated and laid down on Mercator's charts by means of published tables. In the gnomonic projection, on the other hand, the great circles are straight lines, but in this projection it is not possible to present in one map so much as half the surface of the earth. Fig. 23 shows on the equatorial scale of 1 : 250,000,000 the Mercator map of the world between the parallels of 81° N. and 70° S. latitude. The straight line AB represents the uniform compass course, or rhumb-line, between the Gulf of Choronades and the coast of Kamschatka, points for which the latitude and longitude are 42° S., 74° W., and 60° N., 165° E., respectively.

The curved and dotted line between the same points shows the great-circle course. The same great circle is the shortest route between the Falkland Islands and Ceylon and Calcutta. It will be noticed how the great circle from the Falkland Islands runs 10° south, and then turns north on the route to Ceylon; also that the direction of the great circle changes from about 45° west of the meridian at A until it is east and west in latitude 60°. This great circle makes an angle of 60° with the equator. The rhumb-line, or uniform compass course, drawn on the surface of the globe deviates from the great circle much in the same way as on Mercator's Chart the great circle differs from the straight line, but the divergence is, of course, on the opposite side. The great circle from A to B cuts across the narrow peninsula of Alaska, and to make the course possible for a ship a short canal would be required, but this does not affect the geometry, and as no one will ever require to sail from A to B it is of no practical importance.

If a series of templates are cut in card to the curves of the great circles on Mercator's projection corresponding to inclinations of the plane of the great circle to the equator for each interval of, say, 2°, the equator being drawn on each template, great circles can be drawn on the Mercator chart cutting the equator at any longitude, and these templates may be used for drawing approximately the great-circle course between any two points.

ZENITHAL PROJECTIONS

The Gnomonic Projection

Suppose a plane screen to touch the skeleton wire sphere at the North Pole. If the polar axis is horizontal the screen will be vertical. Suppose a shadow picture thrown on the screen by a light at the centre C of the sphere. All the meridians will be straight lines radiating from N, for in Fig. 24 the plane of the paper may be drawn through any meridian. The parallels of latitude will clearly be circles about P as centre. The equator will be projected to infinity, or in other words will not fall on the screen, however large the screen may be. Let us consider how the scale of the map changes for different latitudes. Suppose *a, b, d, f, g,* and *h* to correspond to latitudes 80°, 70°, 60°, 50°, 40°, and 30° respectively, and let A, B, D, F, G, and H be their projections on the screen. Then PA, PB, PD, PF, PG, and PH will be the radii of the parallels of latitude on the projection. The student of trigonometry will recognize that these radii are greater than the radii of the corresponding circles on the sphere as the tangents of 10°, 20°, 30°, 40°, 50°, and 60° are greater than the sines of the same angles, but those not accustomed to trigonometrical ratios may draw the figure and measure the radii. It will be found that if R be the radius of the sphere, PA is 0·176R, PB is 0·364R, PD is 0·577R, PF is 0·839R, PG is 1·192R, and PH is 1·732R. But the radii of the corresponding circles on the sphere are 0·174R, 0·342R, 0·5R, 0·643R, 0·766R, and 0·866R. Hence for latitude 80° the radius of the circle of latitude is increased by 1·15 per cent., for latitude 70° by 6·43 per cent., for latitude 60° by 15·4 per cent., for latitude 50° by 30·48 per cent., for latitude 40° by 55·6 per cent., and for latitude 30° by 100 per cent. For latitude 15° the increase is nearly 300 per cent. On the globe the meridian distances are the

arcs P*a*, P*b*, etc., and these are respectively one-ninth, two-ninths, one-third, four-ninths, five-ninths, and two-thirds of the quadrant of the circle, and their lengths are, therefore, 0·175R, 0·349R, 0·524R, 0·698R, 0·873R, and 1·047R, respectively. On the projection the corresponding polar distances are 0·176R, 0·364R, 0·577R, 0·839R, 1·192R, and 1·732R. Hence the distances from the pole or centre of the map are increased

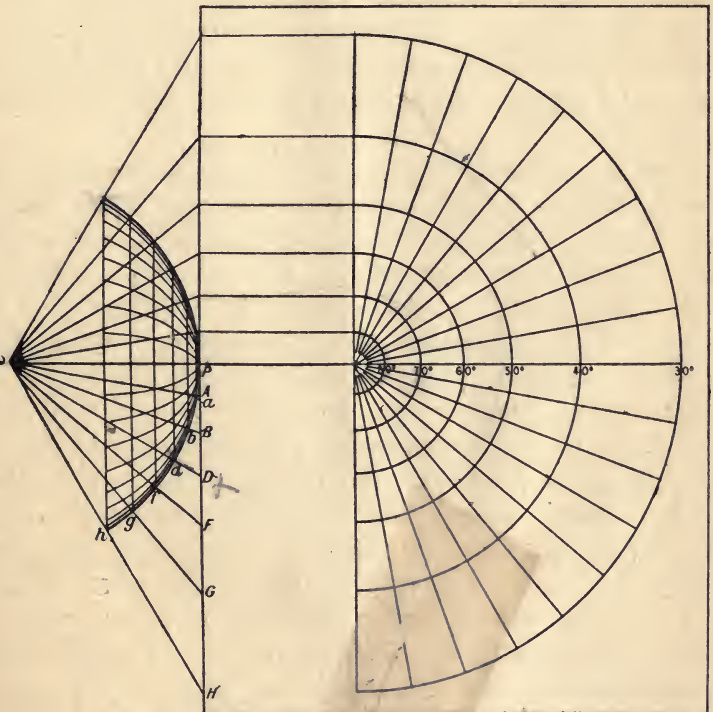


FIG. 24.—POLAR GNOMONIC PROJECTION.

by 0·57, 4·3, 10·1, 20·2, 36·5, and 65·4 per cent. respectively. The meridian *scales*, however, are increased in still greater ratios, for these do not depend on the total increase of the distance from the pole to the circle of latitude, but on the increase in the length of a very small piece of the meridian in the immediate neighbourhood of that circle of latitude. These extensions amount to 3 per cent., 11·3 per cent., 33 per cent., 70·4 per cent., 141·8 per cent., and 300 per cent. respectively.

At latitude 80° the scale of longitude is increased by 1·15 per cent., and the scale of latitude by 3 per cent., at latitude 60° the scales are increased 15·4 per cent. and 33 per cent. respectively, at latitude 45° by 29·3 per cent. and 100 per cent. respectively, and at latitude 30° by 100 per cent. and 300 per cent. respectively, in other words, at latitude 30° the scale of longitude is doubled and the meridian scale is quadrupled. The projection therefore is neither "equal area" nor orthomorphic except near the pole. For a distance of 30° round the pole it makes a fairly good map, and is frequently used for maps of the polar regions. All azimuths from the centre of the map are correct.

If we now suppose the screen to touch the globe at any point other than the pole, and if we draw great circles on the globe through that point to take the place of the meridians, and small circles around that point as pole to take the place of parallels of latitude, and again project from the centre of the sphere, all that we have said about the meridians and parallels of latitude and scales along them will be true of the new set of great circles, which on the map will appear as radii from the centre, and the corresponding small circles which on the map will be circles having the centre of the map as their common centre. Hence this "gnomonic" projection serves fairly well for maps of areas which do not extend to more than about 30° from the centre of the map. The scales vary at different points as explained for the polar map, and since the centre of projection is the centre of the sphere, and therefore in the plane of every great circle, all great circles on the gnomonic projection, whether passing through the centre of the map or not, appear as straight lines.

In Fig. 40, p. 96, the pentagons, other than the central pentagon, afford examples of gnomonic projections on planes touching the sphere at about latitude 26°.

The Stereographic Projection

Let us now move the light to the South Pole, and again placing the screen as tangent plane at the North Pole, obtain the shadowed picture of our skeleton sphere. This will be the

stereographic projection. It is clear that if the screen is large enough the whole sphere can be shadowed upon it except the South Pole itself, which will become the "line at infinity." It is obvious, however, that for practical purposes we cannot project much more than a hemisphere. The rays projected from S at an angle of 45° with the vertical will all pass through points on the equator, and hence it follows that the projection of a point on the equator is at a distance from the pole which is equal to SP, that is, to 2R. It is a well-known proposition that an arc of a circle subtends an angle

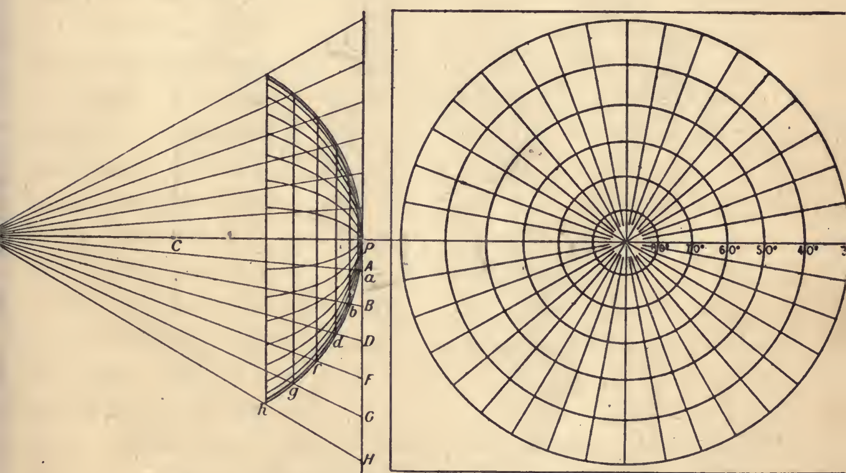


FIG. 25.—POLAR STEREOGRAPHIC PROJECTION.

at the circumference one half of that which it subtends at the centre. Hence, if *a*, *b*, *d*, *f*, *g*, and *h*, as before, are 10°, 20°, 30°, 40°, 50°, and 60° distant from the pole so as to correspond to latitudes 80°, 70°, 60°, 50°, 40°, and 30° respectively, the angles PS*a*, PS*b*, PS*d*, PS*f*, PS*g*, and PS*h*, will be 5°, 10°, 15°, 20°, 25°, and 30° respectively. As the base, SP, of each triangle is 2R, the lengths of the radii PA, PB, PD, PF, PG, PH can at once be found. They are 0·175R, 0·353R, 0·536R, 0·728R, 0·933R, and 1·155R. The radius of the parallel of latitude in the projection corresponding to latitude 20° is 1·400R, to latitude 10° 1·678R, and that of the equator 2R. The meridians are straight lines radiating at equal angles from the

centre of the map, and the projection is, therefore, azimuthal. The radii of the actual parallels of latitude on the sphere are, as before, 0·174R, 0·342R, 0·500R, 0·643R, 0·766R, 0·866R, 0·938R, 0·985R, and R. The percentage errors for the nine circles of latitude are 0·57, 3·22, 7·20, 13·22, 21·80, 33·37, 49·25, 70·35, and 100. These are very much smaller errors than in the case of the gnomonic projection, the error for latitude 30° being only 33·37 per cent. instead of 100 per cent.

We will now attempt a little piece of geometry. Let P, Fig. 26, be any point on the meridian circle, and PT the tangent at P. Let PT meet the tangent at N in T. Join TC and produce SP to meet the tangent at N in K. Then TC bisects the angle PCN. Therefore TCN is equal to PSN, and KPS is parallel to TC. Hence the angle NTC is equal to NKS. But the angles NTC and PTC are equal. Hence PTC is equal to NKS. But since TC is parallel to KS the alternate angles PTC, TPK are equal. Therefore TPK is equal to TKP. Take a very little piece, PR, of the tangent PT, so small that it is indistinguishable from the arc of the circle between the lines RS and PS. Draw RD perpendicular to NS. Then PT may be taken to be equal to the corresponding length of the meridian. Produce SR to meet NK in L, and draw LM parallel to PT. Then the angle LMK is equal to TPK, and therefore to LKM. Hence LM is equal to KL. Now by similar triangles LM is greater than PR as LS is greater than RS or as LN is greater than RD. Therefore, LK is greater than PR, as LN is greater than RD, that is, the little piece of the meridian on the map is stretched in exactly the same

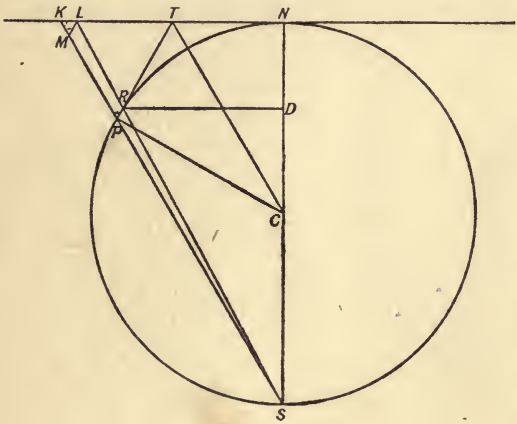


FIG. 26.

proportion as the parallel of latitude at R. This means that the projection is orthomorphic, since the scale is increased equally along the meridians and along the parallels.

All that has been said about the projection on the tangent plane at the pole will be equally true if the tangent plane touches the sphere at any other point, provided that the meridians are replaced by great circles through the point of contact and the parallels of latitude by other small circles having the point of contact as pole. All stereographic projections are therefore azimuthal and orthomorphic and the errors of scale calculated for the polar projection serve equally when any other point is taken as the centre of the map and the angular distances are measured on the globe from that point.

Maps of the World in Hemispheres on this projection are published, the centres of the maps being on the equator in longitude 70° E. and 110° W.

Sir Henry James' Projection

Sir Henry James, as stated above, removed the focus of projection still further from the screen, to a distance of 1·367 times the diameter of the sphere. He found that by this means he could include a portion of the sphere extending 113½° around the centre of the map without excessive distortion. This means 0·6994, or nearly seven-tenths of the whole surface of the globe, and by taking the centre of the map at latitude 23½° N., longitude 15° E., nearly the whole of the British possessions come within the compass of the map.

Sir Henry James' projection is a particular case of a set of projections devised by Colonel Clarke in order to minimise the distortion in zenithal projections. In the stereographic projection both the meridian scale and the scale along the parallels increase as we recede from the centre of the map. In the orthographic projection the meridian scale diminishes. Between these two there are projections for which the total misrepresentation on a map extending over a particular region is a minimum. Colonel Clarke found that for a hemisphere the distance of the focus of projection from the centre of the sphere

should be 1·47R. For a map including a radius of 113½° it should be 1·367R, but for a map including a radius of only 40° it should be 1·625R. These are examples of *minimum error* projections.

THE ORTHOGRAPHIC PROJECTION

When the light is removed to an infinite distance from the screen, so that the projecting rays are parallel, we obtain the orthographic projection. This obviously cannot be employed for more than a hemisphere because the second hemisphere will, in the projection, overlap and coincide with the first. In this case the scale along the circles concentric with the map

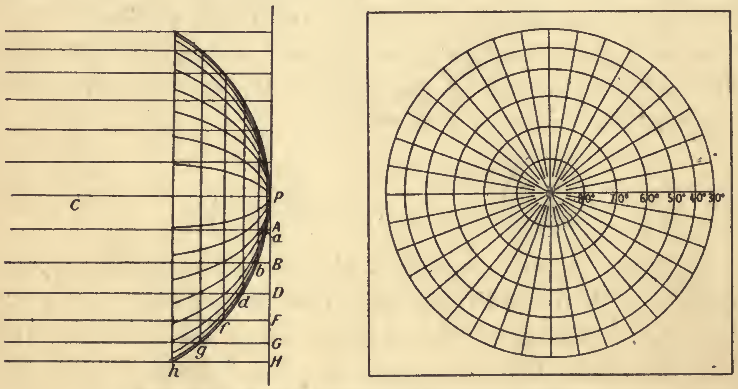


FIG. 27.—POLAR ORTHOGRAPHIC PROJECTION.

is always true, for the corresponding circles on the globe being parallel to the plane of the screen, they are all projected without change of size, but the radial lines are foreshortened more and more as the distance from the centre increases, and at the circumference of the hemisphere the radial scale is zero. The projection, therefore, is useful only within a few degrees of the centre of the map. It is shown in Fig. 27. The orthographic projection on a tangent plane at the equator is shown in Fig. 7, p. 31.

AZIMUTHS

The projections on the tangent plane to the globe, Figs. 24, 25, and 27, all preserve correctly the azimuths of distances measured from the point of contact or map's centre. The map

is symmetrical about the central point just as the "dome of heaven" is symmetrical about the zenith of the observer. Perhaps this is the reason why these projections are called "zenithal."

Non-geometrical Zenithal Projections

The other projections used in map making are not geometrical projections, and like most of the world maps, they cannot be produced by shadows as in the projections just described.

Zenithal Equidistant Projection

Suppose the tangent plane at the pole to be the plane of the map. Draw radii from the pole on the tangent plane to correspond to the meridians. Mark off on one of the radii distances exactly equal to the distances on a meridian of the globe between the parallels of latitude. From the pole as centre describe circles through these points to represent the parallels. The graticule is then complete. It is obvious from the method of construction that distances measured directly from the centre are true, and hence the projection is called the Zenithal or Azimuthal Equidistant Projection. The scale along the parallels, however, increases as the distance from the pole increases inasmuch as the arc measured from the pole to any circle of latitude is greater than the radius of that circle. For 30° from the pole (latitude 60°) the arc is 0·5236R, and the radius of the circle of latitude on

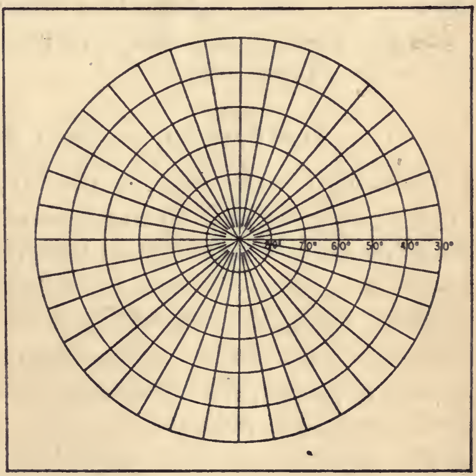


Fig. 28.—Polar Zenithal Equidistant Projection.

the sphere is 0·5R. At 45° the figures are 0·7854R and 0·707R, at 60° (30° latitude) they are 1·0472R and 0·866R, and at 90° they are 1·5708R and R. The projection is not useful for very large areas, but serves well for the polar regions, and is often adopted for them. It is shown for 60° round the pole in Fig. 28.

The method may be adopted when the centre of the map, that is, the point of contact of the map plane and the globe, is any point other than the pole, but some calculation is required to enable the meridians and parallels of latitude, which no longer radiate from or are concentric with the centre of the map, to be drawn. Tables showing the directions and distances from the centre of the map to the points of intersection of the meridians and parallels for different latitudes of the central point have been prepared for this and other zenithal projections, and in all these projections when the centre of the map is not the pole, the meridians and parallels are drawn by laying down their intersections from the published tables which give the distance and direction from the centre of the map for each intersection.

Zenithal Equal-Area Projection

Returning to the map as the tangent plane at the pole, it is clearly possible for the distances of the parallels of latitude on the map to be regulated so that the area enclosed by any parallel is equal to the area of the globe cut off by the same parallel. The projection will then be a Zenithal Equal-Area Projection. The radius of the equator will be $\sqrt{2}$R, so that the area of the circle may be $2\pi R^2$, the same as the area of a hemisphere. The radius of the parallel of 45° will be 0·772R, and the radius of any other parallel may be calculated from the condition that the area of the circle is the same as that of the corresponding polar cap. The radius is always the mean proportional between the diameter of the sphere and the distance measured along the polar axis of the plane of the parallel of latitude from the pole. If the areas of all the circles are equal to the corresponding areas on the sphere, then the area of the ring between any two circles will be the same as that of the corresponding zone on the sphere, and the

projection is, therefore, a zenithal equal-area projection. As the parallels of latitude on the map are all larger than the corresponding parallels on the globe, it follows that the radial scale of the map must be less than that of the globe diminishing as the scale along the parallels increases, since the areas are always equal. The projection is therefore not orthomorphic. It is shown for a hemisphere in Fig. 29.

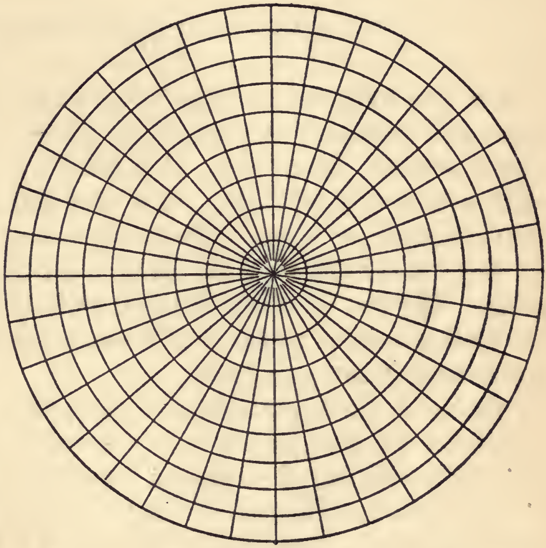


FIG. 29.—POLAR ZENITHAL EQUAL-AREA PROJECTION.

A projection can be obtained with the centre of the map at any point of the globe other than the pole, and the equidistant radii and corresponding circles will take the place of meridians and parallels of latitude. All that has been said respecting the variation of the scale along the meridians and parallels will be true of the scales along equidistant radii, and the corresponding circles when the centre of the map corresponds to any other point of the surface of the globe.

The Zenithal Equal-Area Projection is sometimes known as Lambert's Zenithal Equal-Area Projection.

CONICAL PROJECTIONS

For the mapping of large areas which have considerable extension east and west, like Europe, Asia, North America, and Australia, some form of conical projection is usually employed in the atlas maps and wall maps. The projection will, of course, serve for smaller areas such as political divisions, for the smaller the area the more accurate may the map be made. The test of any system of projection is the extent of the globe's surface which it will represent without more than a certain percentage of error.

We have already considered one purely geometrical projection on a cone. In that case the vertex of the cone represented the pole and the meridian scales increased rapidly as we receded from the circle in which the cone touched the sphere. The method was therefore of little use for the representation of large areas. The methods employed in practice and called "projections," are, like Mercator's and Mollweide's projections, not projections at all in the ordinary geometrical sense.

Simple Conical Projection with One Standard Parallel

Let VPR, Fig. 30, represent a cone touching the sphere NPESQR along the circle of latitude PR. The distances along PR are exactly the same on the cone and sphere. When the cone is laid out flat it will form a sector of a circle, and PR will become a circular arc about V as centre, since all straight lines from V to the circle of contact are equal in length. The sector will be a fraction of the complete circle represented by the ratio of PH to VP. PR is called the standard parallel for the map projection. Now mark off from R, on RV and VR produced, distances equal to the actual distances on the meridian corresponding to 1° or 10° of latitude. These

distances will all be equal, and as the length of VR is greater than that of the arc NR (very much greater when R is near the equator), the point, K, on VR corresponding to latitude 90° will be some distance from V, and the pole itself will be represented by the circle, KH, on the cone, which will develop on the map into an arc of radius VK and of length πKH. The system is therefore not suitable for very high latitudes. The meridians are generating lines of the cone drawn from V to the points in which the circle PR is divided into 36 (or 360) equal parts on the developed map, these lines will be radii of the circular sector spaced at equal angles. The meridian scale

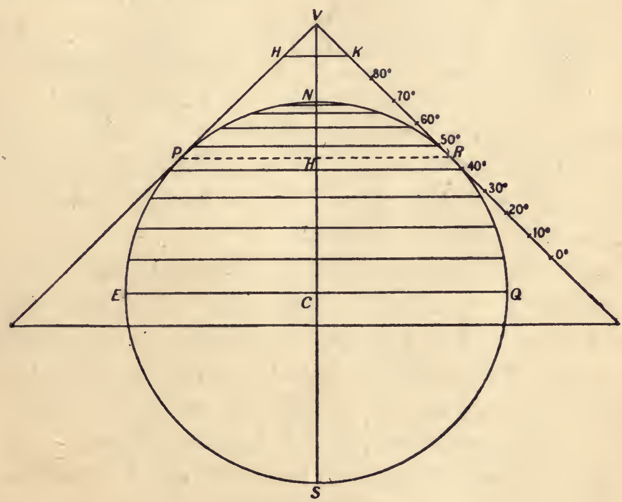


FIG. 30.

will be everywhere correct because the distances have been taken on RV equal to the actual meridianal distances, but the scale along the parallels of latitude will increase as we recede from the standard parallel both north and south, because all the circles on the cone, except PR, are longer than the corresponding circles on the sphere. The projection is therefore neither of equal area nor orthomorphic. It is a compromise, and for 15° of latitude north and south of the standard parallel the errors are not very great. Fig. 31 shows the graticule for the standard parallel of 45°, for

a range of 120° of longitude, that is to say, it shows a third of one hemisphere. While the meridian distances are all exact, as well as the parallel of 45°, the length of the parallel of 30° latitude is 5·605R on the projection while it is 5·441R on the sphere. The length of the parallel of 60° latitude is 3·570R on the projection and 3·1416R on the sphere. This is the simple conical projection with one standard parallel. In a map on this projection, if the map were carried up to latitude 90°, with the parallel of 45° as standard parallel, the length of the arc representing the pole would be 0·953R.

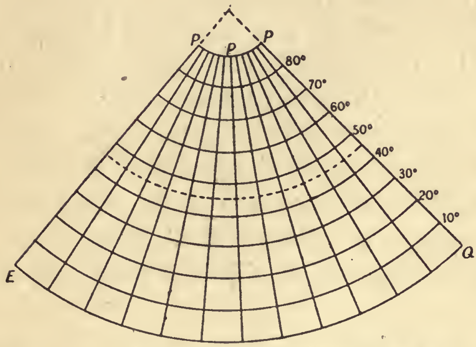


Fig. 31.—Simple Conical Projection with One Standard Parallel.

In maps on this projection the meridians are all straight lines converging to a point above the top of the map, and beyond the north pole, for they are all drawn from the vertex of the cone. The parallels are all equidistant circular areas drawn about the point to which the meridians converge as centres. As the standard parallel is chosen nearer and nearer to the equator the angle of the cone becomes less and less, the meridians become more nearly parallel, the parallels are less curved, and the point of radiation of the meridians moves farther and farther from the map. To draw the projection, therefore, long straight-edges and beam compasses may be required. When the equator is taken as standard parallel the cone becomes a cylinder and the projection becomes the cylindrical equidistant projection (Fig. 32). On this projection the whole surface of the globe is represented by a rectangle $2\pi R$ in length and πR in height. The meridians are all vertical straight lines, the parallels are all horizontal straight lines of equal length and at equal distances, and the poles are as long as the equator. The projection is of very

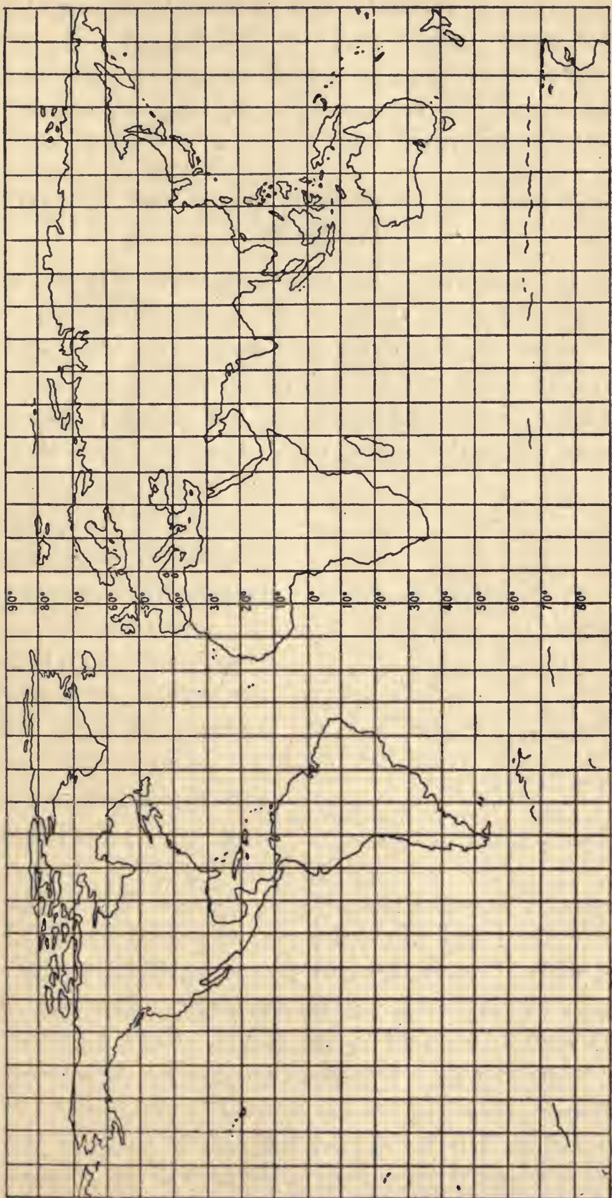


FIG. 32.—CYLINDRICAL EQUIDISTANT PROJECTION.

little use except for the equatorial regions. If the standard parallel be taken nearer and nearer to the pole, the cone becomes more and more obtuse, and at last becomes the tangent plane at the pole when the pole itself is the standard "parallel." The projection then becomes the zenithal equidistant projection already described.

Simple Conical Projection with Two Standard Parallels

While the meridian scale is always correct in the conical equidistant projection, the scale along the parallels is correct only at the standard parallel and increases to the north and south. If we could get two of the circles of the cone equal to two of the circles of latitude on the region to be mapped, there would be two standard parallels on the map along each of which the scale would be correct. Suppose two parallels to be selected corresponding, say, to latitudes 30° and 60°. The radii of these parallels on the sphere are 0·866R and 0·5R respectively. Measure PR, Fig. 33, equal to the meridian arc between latitudes 30° and 60°. Then PR is equal to 0·5236R. Produce RP to V, so that PV is to PR as 0·5 to 0·366 or PV to VR as 0·5 to 0·866. Then the circumference of circles drawn through P and R with V as centre will be proportional to the circles of latitude of 60° and 30° respectively. Let the circular arc ARB be drawn equal to the length of the parallel of 30°, which is to be included between the extreme meridians of the map, AB being bisected by the central meridian VR. Then if the circular arc DF be drawn through P to meet the same meridians in D and F, the length of the arc DF is the length of the parallel of 60° between the same meridians. Divide PR into 3 (or 30) equal parts, and draw circles about V through the

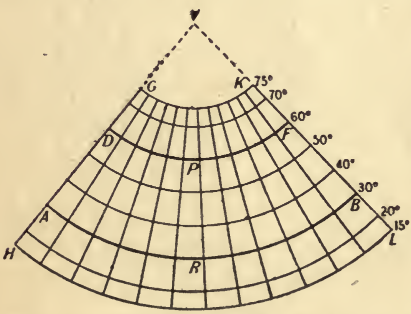


Fig. 33.—Simple Conical Projection with Two Standard Parallels.

points of division. Produce VA, VB and VR, and continue the equal divisions above and below P and R as far as the map is to extend, say 15° north and south from P and R respectively. Also divide the arc AB to correspond to intervals of longitude of 10° (or 1°), and draw radii from C through the points of division. The complete map between longitudes corresponding to VH and VL and latitudes corresponding to GK and HL will then be included in the figure GKLH.

Distances along the meridians are true by construction, as are also the distances along the two standard parallels DF and AB. North of DF and south of AB the scale along the parallels increases as we recede from the standard parallels. Between the standard parallels the central meridian is represented by the straight line DF. But on the globe the portion of the meridian is not straight but bulges outwards from the chord joining the two points on the meridian. Hence, between the standard parallels the parallels on the globe are longer than those on the projection or the scale along the parallels is reduced in the map. It is clear, however, that as the errors are reduced to zero on two parallels which may be so chosen as to cut the central meridian at the first and third quarters, the errors at the centre and at the top and bottom of the map are much less than they would be if the cone corresponded with the sphere at the central parallel only and the generating lines of the cone, the meridians on the map, receded from the spherical surface throughout the whole distance from the central parallel to the top and bottom of the map. This projection is known as the Conical (Equidistant) Projection with two standard parallels. The meridians are straight lines converging to the vertex of the cone, the parallels are equidistant circular arcs drawn round the vertex of the cone as centre, and the pole is also a circular arc of finite length. The projection is neither equal area nor orthomorphic, but for continents situated like Europe, Asia, North America, or Australia it serves well for atlas maps. The cone on which the map is projected cannot be made to cut the globe in the two standard parallels, because the distance between the parallels on the cone is the length of the arc of the

meridian, and on the globe the straight line distance is only the chord of that arc.

Conical Equal-Area Projection

It is possible so to modify the conical projections with one or two standard parallels as to make the distance between successive parallels greater or less than the true meridian distances in "proportion" as the lengths of the parallels are less or greater than their true lengths on the sphere. The projection is then a conical equal-area projection. The pole is in this case the vertex of the cone, and as the angle of the developed cone is less than four right angles, the meridian scale must be increased towards the pole. This is one of Lambert's projections.

Conical Orthomorphic Projection

If the meridian distances on the projection are chosen so that they are increased or diminished in the same ratio as the corresponding parallels, and not in the inverse ratio, the projection becomes orthomorphic. The pole is necessarily a point in an orthomorphic projection and is the vertex of the cone. The scale increases north and south from the standard parallel or parallels, and if there are two standard parallels the scale is reduced between them. This projection is Lambert's second projection. In an orthomorphic projection the scale must be the same at any point along both meridians and parallels, for length and breadth must be increased in the same ratio if the shape is to be preserved.

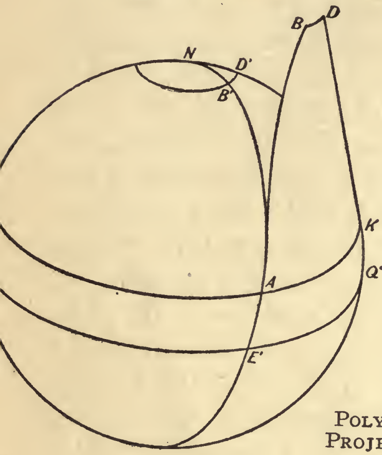
The Polyconic Projection

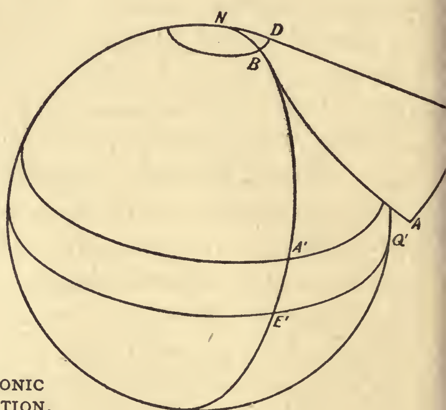
To understand the following conical projection, known as the polyconic, a considerable exercise of the imagination will be required. Take a flexible screen of sufficient length and breadth to include the map which is to be made and bend it into a portion of a cone so as to touch the parallel of latitude which is to be the lower boundary of the map and draw the parallel on the bent screen, marking the meridians. The

sheet must not be sufficiently large to extend nearly to the vertex of the cone or the bending to follow will not be possible. Now roll the conical surface up the central meridian, allowing the cone to open out until it is a tangent cone at the next parallel of latitude. Draw that parallel and mark the divisions upon it corresponding to the meridians. Proceed in this way, rolling the central meridian of the map up the central meridian on the globe, and bending the cone so as to touch the globe along each parallel of latitude until the whole range of latitude to be included in the map has been covered. Then draw the meridians through the points of division of each parallel of latitude. The result will be a projection in which every parallel is a standard parallel, and the scale will therefore be true along all the parallels. It is also true along the central meridian, for this has rolled without slipping on the meridian of the globe. The parallels will be circular arcs, but each drawn about a different centre, namely, the vertex of the cone when its angle was arranged so that the cone was a tangent to the sphere along that parallel. The centres of the circles of latitude will therefore come nearer to the centre of the map as the latitude is increased. Since the distances on the central meridian are all equal, it follows that the distances between the parallels will increase as we move east and west from the central meridian. The meridian scale, therefore, is not the same in different portions of the map, but as every parallel is a standard parallel the scale along every parallel is true. The map is therefore neither orthomorphic nor an equal-area projection, but from the process of construction it very nearly fits the sphere over a considerable area. If the extent of the map in longitude is very great there will be a great deal of slipping on meridians remote from the central meridian as the cone is readjusted to the suitable angle for touching the sphere along the successive parallels, and this means distortion and inequality of area towards the margin of the map. For comparatively small areas the system affords a very high degree of accuracy, but the meridians are all curved and convex towards the sides of the map, the curvature being slight near the central meridian but increasing somewhat rapidly after the first 30° of longitude

on each side of the centre. It is, therefore, not possible to fit together maps of tracts of country adjoining east and west when drawn on this system, each with its own central meridian.

Figures 34A and 34B are conventional drawings, not true projections, representing the portion of the tangent cone, or





POLYCONIC PROJECTION.

FIG. 34A.

FIG. 34B.

mantle, ABDK, corresponding to a map projection extending over 120° of longitude from latitude 15° to latitude 75°. Fig. 34A represents the mantle in its lowest position, touching the globe along the parallel of 15°. The vertex of the cone in this case is at a distance of 2·85R from the pole measured along the polar axis produced, R being the radius of the globe. Fig. 34B represents the same mantle when the central line, KD, has rolled along the meridian KD′ until D has come in contact with the sphere at D′ and the conical mantle has opened out so as to touch the sphere along the parallel of 75°. As there is pure rolling with

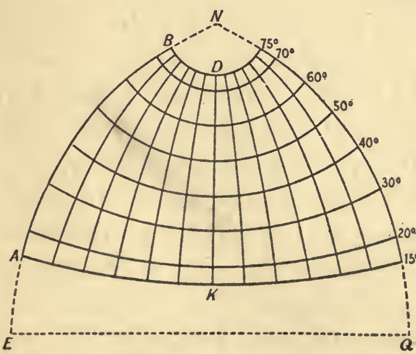


FIG. 34C.—POLYCONIC PROJECTION.

no slipping between KD and the meridian of the globe the straight line KD is equal to the arc of 60° on the meridian, but this is the only line on the mantle which rolls without slipping as the cone expands. In the position shown in Fig. 34B the vertex of the cone is only ·035R above the pole N.

Fig. 34C shows the corresponding map projection when the conical mantle has been fully developed, as would be the case if the rolling were continued until it became the tangent plane at the pole N. The full lines show the map projection with the meridians and circles of latitude between the parallels of 15° and 75°. The dotted lines extend the outlines of the map projection to the pole and the equator. South of the equator the projection is a reduplication of that to the north. If the meridians and parallels were drawn upon the sphere with printer's ink they would be printed off on the expanding conical mantle as successive portions of the mantle touched the sphere and would form the polyconic projection when the mantle was laid flat. It will be seen that this system lends itself very well to a map which does not extend much beyond 30° on each side of the central meridian and does not extend within 20° of the pole. The reader can easily draw for himself the projection for the whole hemisphere or the whole world. The circles of latitude (and the straight line of the equator) shown in Fig. 34C have only to be continued so that their length is three times that shown in the figure. The reader would do well to complete this projection and compare it with the sinusoidal projection shown in Fig. 14.

In Fig. 34C the centres of the several circles of latitude are the vertices of the corresponding cones which touch the globe along those circles.

The Rectangular Polyconic Projection

The rectangular Polyconic projection is a modification of the simple Polyconic. The central meridian and the circular "parallels" are drawn as for the simple Polyconic, but instead of measuring equal distances along the parallels for the points of intersection of the meridians the meridians are drawn so as to cut all the parallels at right angles. The meridians

may be spaced so as to give a true scale along the equator or along any other parallel. This projection has been much used by the War Office.

The International Map

The Polyconic projection was originally recommended for the International Map on the scale of 1 to 1,000,000, which has been mentioned near the beginning of this little book, but in 1909 the Committee adopted a modified form of the projection involving another compromise with a view to securing that adjacent maps should fit along their margins. In this form the top and bottom parallels are drawn as circles with different centres. They are equally divided for degrees of longitude, and the meridians are drawn as straight lines through these points of division. It has been pointed out that in the Polyconic projection the meridian scale increases as we go east or west from the central meridian, while upon the central meridian it is true. In the International Map each map covers four degrees of latitude and six of longitude. The extreme parallels of latitude are drawn a little nearer to one another than in the Polyconic projection, so that the distances between the extreme parallels along the meridians 2° E. and 2° W. of the central meridian may be true distances, and it is these meridians which are equally divided for the positions of the parallels. The parallels are constructed from tables which allow for the departure of the figure of the earth from an exact sphere (see p. 105).

Bonne's Equal-Area Projection

There is one other map projection which is much used in atlases and is known as Bonne's. The Sanson-Flamsteed Sinusoidal projection is the particular case of Bonne's projection when the equator is the standard parallel. It is a modified conical projection. The angle of the cone is determined so that the surface may touch the globe along the standard parallel. The parallels are all drawn as concentric circles through points dividing equally the central meridian, as in the case of the simple conical projection, and each is made of the exact

length of the corresponding parallel on the globe, and equally divided for the several meridians. The scale is therefore correct along each parallel, and as the parallels are concentric circles and the central meridian is truly divided the scale is correct in directions perpendicular to the parallels at all points of the map. The projection is therefore an equal-area projection. It differs from the Polyconic projection since all the circles of latitude are concentric. The meridians are curved and make angles with the parallels, which differ more and more from right angles as they diverge from the central meridian. Hence the projection is not orthomorphic. It serves well for areas not near the poles and not extending through any great range of longitude. It is used in many atlases for maps of continents. The particular case of the Sanson-Flamsteed Sinusoidal projection, when the equator is the standard parallel, has already been discussed at length. The projection is shown in Fig. 35 for a space of 120° of longitude between the equator and the pole.

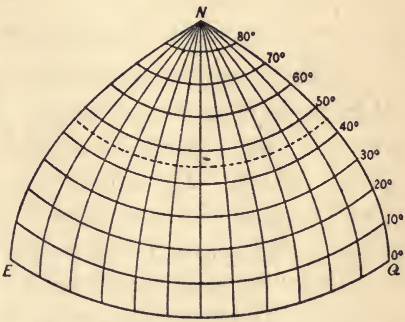


FIG. 35.—BONNE'S EQUAL-AREA PROJECTION.

OTHER PROJECTIONS

The Polyhedric Projection

There is another projection to which reference should be made though it is really only an orthographic projection of different small portions of the earth's surface on separate picture planes. The reader is probably familiar with the way in which gems are cut to show "facets." If the original surface of the gem is curved but symmetrical, in order to save waste in cutting, the gem-cutter arranges that all the angles of the cut gem shall lie in the original surface. By this arrangement no more material is cut away than is necessary to render each facet perfect. Suppose a sphere to be divided into any number of zones by parallel circles (parallels of latitude). Between each pair of parallels any number of equal facets may be cut, and if each zone is divided into the same number of equal parts by the same meridians as guides for the cutting, all the facets in each zone will be equal quadrilaterals diminishing towards the poles and replaced by triangles at the polar caps, but the facets may be cut in any other way. The figure so produced has many sides or faces, and is consequently called a polyhedron. Suppose any such polyhedron to be inscribed within the terrestrial globe. From all the points in the outlines of any area of the globe draw perpendiculars to the corresponding facet. A series of maps will thus be obtained on the facets of the polyhedron. There will be small areas of the globe opposite the edges of the polyhedron which will not be projected on any face, but if the planes of the faces are produced till they actually cut the surface of the globe the whole surface will be represented on the series of maps, and some small portions will appear on each of two adjacent maps. Maps drawn on this system are known as polyhedric projections. They are very simple in character, and correspond to the central portion

of the orthographic projection described on p. 31. For small areas they are sufficiently accurate, the central portion of the map being true to scale in all directions. For large areas the criticisms made respecting the orthographic projection hold good.

THE FIVE REGULAR SOLIDS

There are five, and only five, regular solids, that is, solids of which all the angles and all the faces are exactly alike. If a solid angle is to be formed out of three or more plane angles the plane angles together must make up less than 360°, for if they make exactly 360° the result of fitting them together will be a plane and not a solid angle like the angle of a cube. The angle of a hexagon is 120°, and three hexagons, therefore, fit together in a plane and do not form a solid angle. In a polygon of more than six sides the angles are greater than 120°, and three of them cannot be fitted together at all. To make a regular polyhedron it is therefore necessary that the faces should not have more than five sides, and we are limited to pentagons, squares, and equilateral triangles out of which to build our figure. Three pentagons may be brought together to form a solid angle, as the three angles together make only 324°. Four squares will make a plane, but three squares will make the angle of a cube. The angle of an equilateral triangle is 60°. Six would fit together in a plane, but five, four, and three would make solid angles. The cube, the faces of which are six squares, is sufficiently well known to require no description. It is a favourite exercise to make a little basket out of six regular pentagons. One is taken for the base, five others are fastened one to each side of the base (Fig. 36). Then these five are bent up towards one another till the edges meet, and thus a basket

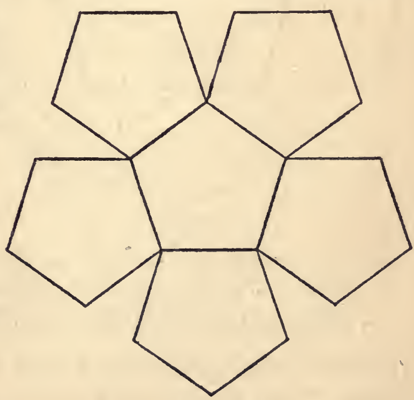
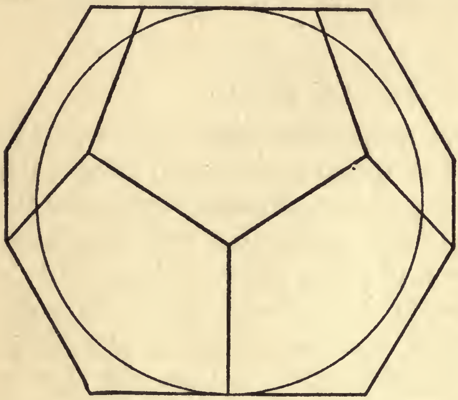
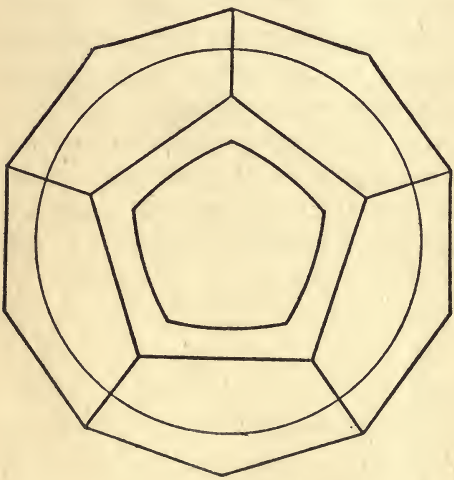


FIG. 36.

is formed, the rim of which appears with five elevations and five depressions. If two such baskets are fitted together, one being inverted on the other so that the projecting points in the rim of the one fits into the depressions in the rim of the other, a regular dodecahedron will be formed, all its faces consisting of equal pentagons and all its angles being formed by the meeting together of the angles of three pentagons. The solid is shown in elevation and plan in Fig. 37.



Elevation.

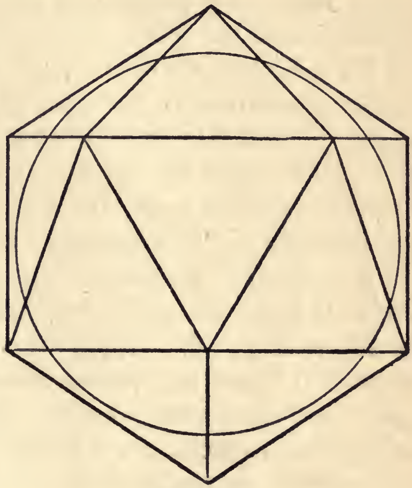


Plan.

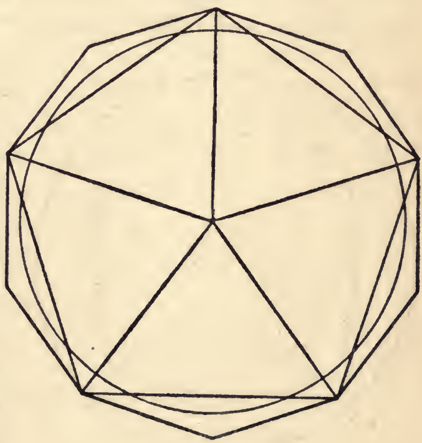
FIG. 37.—ELEVATION AND PLAN OF REGULAR DODECAHEDRON, SHOWING INSCRIBED SPHERE AND CURVILINEAR PENTAGON COVERING ONE-TWELFTH OF SPHERE.

If three equilateral triangles meet to form a solid angle a triangular pyramid is formed of which the base is a fourth equilateral triangle. These four triangles make the tetrahedron. If four equilateral triangles meet to form a solid angle, they together constitute the four sides of a square pyramid. If a second precisely equal pyramid be joined the two pyramids will form a regular solid, known as the octahedron, bounded by eight equilateral triangles, and each of the six solid angles will be formed by the meeting of four equilateral triangles. If five equal equilateral triangles unite to form a solid angle they will form the faces of a

pentagonal pyramid. To each of the bases another equal triangle may be attached so as to hang with its vertex downwards. If a second set of ten triangles be connected together in this way and one inverted on the other, it will be possible to adjust to two sets of five triangles so that they fit into one another like the triangular tops of the pentagons when the dodecahedron was made. A regular solid is thus formed with twenty equilateral triangles, and each of its solid angles is constituted by the meeting of five equilateral triangles. It is known as the icosahedron, or twenty-sided figure (Fig. 38).



Elevation.



Plan.

FIG. 38.—ELEVATION AND PLAN OF REGULAR ICOSAHEDRON, SHOWING INSCRIBED SPHERE.

GNOMONIC PROJECTIONS ON POLYHEDRA

A regular polydehron may be inscribed in a sphere, so that all its angular points are on the surface of the sphere, or it may be circumscribed about the sphere so that all its faces touch the sphere at their central points. In shadow projections we have necessarily taken the screen to be outside the sphere, and it will be convenient to suppose the sphere to be inside the polyhedron on the faces of which it is to be projected. If we then project

the sphere on the polyhedron by rays proceeding from inside the sphere, since any ray which cuts the sphere will, if produced, cut the polyhedron and cut it only once, and any ray which meets the polyhedron must perforce cut the sphere, it follows that the whole surface of the sphere will be projected on the polyhedron with no gaps as in the case of the orthographic projection on the faces of a polyhedron, and no overlapping. First take the case of the cube. Let the poles of the globe be touched by the top and bottom faces of the cube. Then the vertical faces will touch the equator at four points separated by 90° of longitude. Let one of these points be on the meridian of Greenwich. Then the other points of contact will be in longitude 90° E., 180° and 90° W. The planes of the meridians of 45° E. and 45° W. will pass through the vertical edges of the cube and planes drawn through the centre of the sphere and the horizontal edges of the cube will cut the sphere in great circles inclined 45° to the equator. Hence if the maps be projected on the faces of the cube from the centre of the sphere the six maps so formed will correspond to the portions of the sphere comprised between the arcs of four great circles, each inclined 45° to the horizon and passing through the horizontal edges of the cube, and two meridian circles at right angles to each other and passing through the vertical edges of the cube. The six figures into which the surface of the sphere is thus divided are precisely similar and equal. Each is bounded by four arcs of great circles (Fig. 39). The great circles meet three and three together at the common angular points, and since all the angles are equal each angle between two great circles is 120°. In the projection the six curvilinear figures become six squares, so that these angles of 120° are projected

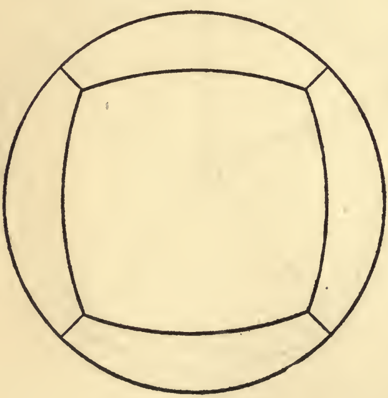


Fig. 39.

into angles of 90°. This affords a measure of the distortion at the angles of the cube where the distortion of each map is greatest. Moreover, the area of each map is $4R^2$, but the area of one sixth of the sphere is $\frac{\pi}{6}4R^2$, or ·5236 of $4R^2$, so that the area of each map is nearly double that of the portion of the sphere represented, although the scale is true at the centre of the map. The plane of any great circle of the sphere passes through the centre of the sphere and cuts any faces of the cube which it meets in straight lines. Hence any great circle is represented by straight lines on the maps all lying in one plane. It will be a useful exercise to make a cardboard cube, to draw the maps on the several faces and to trace two or three great circles, the planes of which do not pass through the edges of the cube.

If the sphere be placed inside a regular dodecahedron with its poles in contact with two parallel faces the plane of the equator will cut the other ten faces symmetrically and the gnomonic projection of the equator on the dodecahedron will be a regular decagon. Each pentagonal face will receive the projection of one-twelfth of the surface of the sphere, so that the pentagonal maps will each represent exactly one-half of the area represented on one face of the cube. The great circles of the sphere, the planes of which pass through the edges of the dodecahedron, form the boundaries of the areas on the sphere corresponding to the several maps. As in the case of the cube the circles meet three and three together on the sphere, so that they always meet at angles of 120°. But the angle of a regular pentagon is 108°, so that at the angular points of the pentagonal maps, the points where the distortion is greatest, the angle of 120° on the sphere is represented by 108°, whereas in the case of projection on the cube it was represented by only 90°. As in the case of the cube all great circles on the sphere will be represented by straight lines on each map, and any great circle will be represented by a series of straight lines crossing certain of the maps so as to form a continuous course round the dodecahedron. The projection of meridians and parallels for intervals of 9° over half the earth's surface is

shown in Fig. 40. On ten of the twelve faces the parallels, except the equator, are hyperbolas. All the meridians are represented by straight lines.

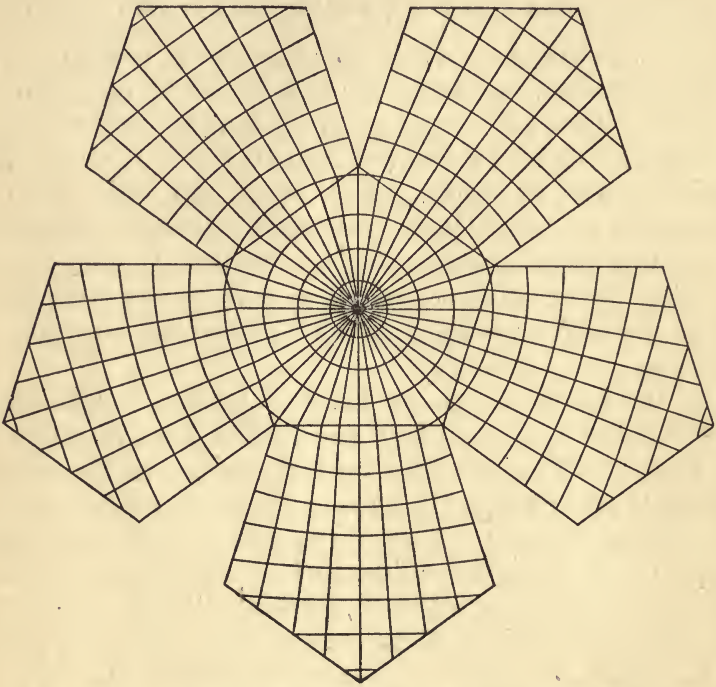


FIG. 40.—GNOMONIC PROJECTION ON DODECAHEDRON.

Maps formed by projection on the faces of a tetrahedron or octahedron would have nothing to recommend them and to compensate for the inconvenience of triangular maps, and even in the case of the icosahedron, although the area of the sphere is divided into twenty equal parts, the shape of these parts deviates so much from the circle that the distortion at the angles is greater than in the case of the dodecahedron, which gives the best set of projections of any of the regular polyhedra.

OTHER METHODS OF PROJECTION

Other methods of projection based on compromises between errors of form and errors of dimensions have been occasionally

adopted, but they involve somewhat difficult mathematical investigations, and they are not to be found in ordinary atlases. One such projection was designed by Sir George Airy so as to reduce the sum of the squares of the two errors to a minimum all over the map. Other examples of minimum error projections are Colonel Clarke's zenithal projections referred to above.

BRITISH ORDNANCE MAPS

Some of the British Ordnance Maps, namely the 1-inch map of England and the 6-inch map of the United Kingdom, are made on Cassini's rectangular co-ordinate system. A point O, Fig. 41, is selected for the centre of the map, and a central meridian is laid down. To find the position on the map of the point P, a great circle PM is supposed to be drawn from P perpendicular to the central meridian, meeting it in M. The actual lengths of OM and PM on the earth's surface are then calculated by spherical trigonometry, and the distances OM and MP are laid off vertically and horizontally on the map. The map is then constructed as one sheet, although the total length from north to south of the 6-inch map of the United Kingdom is something like 300 feet, and is printed in a great many sheets. As, however, all the sheets are parts of one map they can be all put together into one sheet, and will fit at the edges, if we have a table large enough to contain them. It was pointed out that separate polyconic maps of adjacent areas will not fit at the edges, and that the system has been modified for the International Map to meet this difficulty. The errors introduced into the Ordnance Maps are very small over the United Kingdom, and the curvature of the meridians is not appreciable on a single sheet, but the system is not applicable to a whole continent.

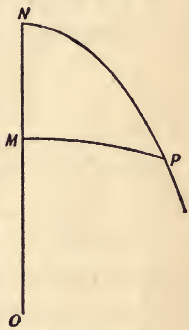


FIG. 41.

PUBLISHED ATLAS MAPS

The projections most commonly employed for Atlas Maps are—

Bonne's Equal-Area Projection for all the continents, except that in the case of Africa and South America the Sanson-Flamsteed Sinusoidal Projection is employed, but this is only a particular case of Bonne's with the equator as standard parallel.

The Zenithal Equidistant for Asia, Africa, North America, South America, and Oceania.

The Zenithal Equal Area for Europe, Asia, Africa, North America, and Oceania.

The Sanson-Flamsteed Sinusoidal projection for Africa, South America, Australia, and Oceania.

The Simple Conic (equidistant parallels) for Europe.

The Conical Orthomorphic for Europe and Australia.

Mercator's Projection for Oceania.

The Choice of a Projection

The projection to be selected for any map depends not only on the extent of country to be mapped, but also on the special purpose for which the map is mainly required. For statistical purposes, the representation of the distribution of population, of minerals or vegetation, or other forms of wealth, for the distribution of rainfall, and many other purposes, Equal-Area projections are the best, and for whole-world maps Mollweide's projection is frequently employed, but sometimes the Cylindrical Equal-Area projection is preferred. When it is important that all great circles should be represented by straight lines, the central, or gnomonic, projection must be employed. Mercator's projection has been largely used for ocean charts, on account of the readiness with which the uniform compass course, rhumb-line, or loxodrome, can be laid down, but great circle courses must be calculated. The writers of the article on Map Projection in the *Encyclopædia Britannica* state that "for conveying good general ideas of the shape and distribution of the surface features of a continent or hemisphere *Clarke's perspective* projection is the best." This projection, in order to reduce the errors of distortion to a minimum, is taken from a point distant 1·47 radii from the centre of the sphere when the map is to include a whole hemisphere, but the map

may be made to include an area greater than a hemisphere by reducing the distance of the point of projection from the centre (see p. 73). The writers in the *Encyclopædia Britannica* continue:—" For exhibiting the progress of polar exploration the *polar equidistant* projection should be selected. For special maps for general use on scales of 1/1,000,000 or smaller, and for a series of which the sheets are to fit together, *the conical, with rectified meridians and two standard parallels*, is a good projection. For topographical maps in which each sheet is published independently, and the scale is not smaller than 1/500,000, either form of polyconic is very convenient." It will be noticed that, as in the Polyconic projections the meridians are always curved outwards from the central meridian, it is not possible to fit together two adjoining sheets drawn with separate central meridians. A Polyconic projection in several sheets can only be fitted together if all the sheets are drawn from the same central meridian.

One of the simplest projections for small areas is a modification of the Polyhedric projection. Four points are taken at the intersection of two meridians and two parallels which are not more than a few degrees apart. Through these four points a plane can be drawn, and the features of the map are projected orthogonally from the globe upon this plane by simply drawing perpendiculars to the plane. As pointed out in the account given of the Polyhedric projection, maps so drawn cannot be fitted together into a larger sheet.

For Africa and South America, which lie on the equator and do not extend over a wide range of longitude, the Sanson-Flamsteed (Sinusoidal) projection is very suitable, and is largely used. It is sometimes used for Australia. For Europe, Asia, and North America Bonne's projection is suitable, or the Zenithal Equidistant or Zenithal Equal Area, but it must be remembered that the simple Conical projections, with one or two standard parallels, have the advantage of straight meridians, and, as the parallels are concentric circular arcs all equally divided, the scales are the same on each meridian, and at all points of the same parallel.

The Identification of Projections

Mr. Hinks gives the following suggestions for identifying the projections described in this book:—

1. Parallels concentric circles.
    - (*a*) Meridians curved—Bonne.
    - (*b*) Meridians straight.
        - (i) Parallels equidistant—Simple conic with one or two standard parallels.
        - (ii) Distance between parallels decreasing towards the pole—Conical Equal Area.
        - (iii) Distance between parallels increasing towards the pole—Conical Orthomorphic.
2. Parallels straight lines.
    - (*a*) Meridians curved, and
        - (i) Parallels equidistant—Sanson-Flamsteed.
        - (ii) Parallels closer towards the poles—Mollweide.
    - (*b*) Meridians straight lines, and
        - (i) Parallels equidistant—Simple Cylindrical.
        - (ii) Parallels closer towards the poles—Cylindrical Equal Area.
        - (iii) Parallels wider apart towards the poles—Cylindrical Orthomorphic (Mercator).
3. Meridians straight lines and parallels curved (other than polar projections)—Gnomonic.
4. Both meridians and parallels curved.
    - (*a*) Cutting at right angles—Zenithal Orthomorphic or Rectangular Polyconic.
    - (*b*) Not cutting at right angles—
        - (i) Central Meridian divided equally by parallels —Zenithal Equidistant.
        - (ii) Divisions on Central Meridian decreasing from the centre—most probably Zenithal Equal Area.

APPENDIX

Note on Mercator's Projection

The radius of the parallel of latitude θ on a sphere of radius r is $r \cos \theta$. Hence, if a degree of longitude in latitude θ is to be made equal to a degree at the equator its length must be divided by $\cos \theta$. If the scale of the map is to be increased in all directions in the same ratio, then the length of the degree of latitude measured along the meridian must also be increased in the same ratio. Hence, if the latitude be measured in radians and y be the distance of the parallel of latitude θ from the equator on the Mercator map corresponding to a sphere of radius r,

$$\frac{dy}{d\theta}=\frac{r}{\cos \theta}, \text{ and}$$

$$y=r\int_0^\theta \frac{d\theta}{\cos \theta}$$

$$\text{But } \int \frac{d\theta}{\cos \theta}=\log_e \cot \left(\frac{\pi}{4}-\frac{\theta}{2}\right),$$

which is zero when $\theta=0$. Hence,

$$y=r \log_e \cot \left(\frac{\pi}{4}-\frac{\theta}{2}\right)$$

or, if the latitude is L°,

$$y=r \log_e \cot (45-\tfrac{1}{2}L)^\circ.$$

Remembering that $\log_e N=2\cdot302585 \log_{10} N$ this expression can be used at once for determining the distance of any parallel of latitude from the equator. The length of a degree at the equator is $\frac{\pi r}{180}$.

In this investigation no allowance is made for the ellipticity of the earth.

If a great circle is inclined at an angle α to the plane of the equator, and cut in latitude L, a meridian the longitude of which, measured from one of the nodes, is λ, then,

$$\tan L=\tan \alpha \,.\, \sin \lambda$$

or

$$\sin \lambda=\cot \alpha \,.\, \tan L.$$

From these equations the latitude can be determined at which the great circle cuts any meridian or the longitude at which it cuts any parallel of latitude. Either equation or both equations may therefore be used to trace the course of a great circle on any map projection when the inclination of the circle to the equator and the position on the equator of one of the points of section are known. The equations may be used for finding points on each meridian, or each parallel of latitude or both on Mercator's map, and this is the way in which the great circle was traced in Fig. 23.

If we substitute for L in the equation $y=r \log_e \cot\left(\frac{\pi}{4}-\frac{L}{2}\right)$ the expression $\tan^{-1}\tan\alpha\sin\lambda$, and remember that, if x be the linear distance measured along the equator corresponding to a difference of longitude λ, $x=r\lambda$, or $\lambda=\frac{x}{r}$, we obtain as the equation to a great circle on Mercator's projection :—

$$y=r\log_e \cot\left(\frac{\pi}{4}-\frac{1}{2}\tan^{-1}\sin\frac{x}{r}\tan\alpha\right),$$

an equation which no one would use for any purpose whatever.

In the following Table, Column I shows for each degree of latitude the scale on Mercator's projection in terms of the scale at the equator, that is the scale of the equivalent globe, on the assumption that the earth is spherical. Column II shows the same allowing for the ellipticity of the meridian. While the figures in Column I are simply the secant of the latitude, those in Column II are the value of $\left(1+\frac{b^2}{a^2}\tan^2 L\right)^{\frac{1}{2}}$ where a represents the equatorial and b the polar semi-diameter. Column III shows, in terms of the length of a degree of longitude on the equator, the length of a degree of latitude on Mercator's scale, for each degree allowing for the earth's ellipticity. The ratio of the actual length of a very small fraction of a degree of the meridian in any latitude to the length of the same fraction of a degree measured along the equator is

$$\frac{ab^2}{(a^2\cos^2 L+b^2\sin^2 L)^{\frac{3}{2}}},$$

where L represents the latitude. Column III is obtained from Column II by multiplying by this factor, or from Column I by multiplying by $\frac{b^2}{a^2\cos^2 L+b^2\sin^2 L}$. In high latitudes the value of the ratio changes so much for one degree that accuracy demands that much smaller intervals than a degree should be taken. This column has, therefore, not been carried beyond latitude 85°. The

actual distance on the map of any parallel of latitude, can be found by adding all the figures in Column III up to that parallel, and multiplying the result by $\frac{\pi a}{180}$, where a is the radius of the equator corresponding to the scale of the map, or $\frac{\pi a}{180}$ is the length of a degree of longitude measured along the equator.

TABLE I.

Lat.	I.	II.	III.	Lat.	I.	II.	III.	Lat.	I.	II.	III.
0°	1·000	1·000	0·993	31°	1·167	1·166	1·161	61°	2·063	2·058	2·060
1°	1·000	1·000	0·993	32°	1·179	1·178	1·173	62°	2·130	2·125	2·127
2°	1·001	1·001	0·994	33°	1·192	1·191	1·186	63°	2·203	2·198	2·200
3°	1·001	1·001	0·994	34°	1·206	1·205	1·200	64°	2·281	2·275	2·278
4°	1·002	1·002	0·995	35°	1·221	1·220	1·215	65°	2·366	2·360	2·364
5°	1·004	1·004	0·997	36°	1·236	1·235	1·231	66°	2·459	2·452	2·456
6°	1·006	1·006	0·999	37°	1·252	1·250	1·247	67°	2·559	2·552	2·557
7°	1·008	1·008	1·001	38°	1·269	1·267	1·264	68°	2·670	2·662	2·667
8°	1·010	1·010	1·003	39°	1·287	1·285	1·282	69°	2·790	2·782	2·787
9°	1·012	1·012	1·005	40°	1·305	1·303	1·300	70°	2·924	2·915	2·922
10°	1·015	1·015	1·008	41°	1·325	1·323	1·320	71°	3·072	3·063	3·069
11°	1·019	1·018	1·012	42°	1·346	1·344	1·341	72°	3·236	3·226	3·234
12°	1·022	1·022	1·016	43°	1·367	1·365	1·362	73°	3·420	3·410	3·418
13°	1·026	1·026	1·021	44°	1·390	1·388	1·385	74°	3·628	3·617	3·626
14°	1·031	1·031	1·025	45°	1·414	1·412	1·409	75°	3·864	3·853	3·862
15°	1·035	1·035	1·029	46°	1·440	1·437	1·435	76°	4·134	4·122	4·133
16°	1·040	1·040	1·034	47°	1·466	1·463	1·461	77°	4·445	4·431	4·443
17°	1·046	1·046	1·040	48°	1·494	1·491	1·490	78°	4·810	4·795	4·809
18°	1·051	1·051	1·046	49°	1·524	1·521	1·520	79°	5·241	5·225	5·241
19°	1·058	1·058	1·052	50°	1·556	1·553	1·552	80°	5·759	5·742	5·759
20°	1·064	1·064	1·058	51°	1·589	1·586	1·585	81°	6·392	6·373	6·392
21°	1·071	1·071	1·065	52°	1·624	1·621	1·620	82°	7·185	7·161	7·183
22°	1·079	1·079	1·073	53°	1·662	1·658	1·658	83°	8·206	8·180	8·205
23°	1·086	1·085	1·080	54°	1·701	1·697	1·698	84°	9·567	9·535	9·566
24°	1·095	1·094	1·089	55°	1·743	1·739	1·739	85°	11·474	11·436	11·473
25°	1·103	1·102	1·098	56°	1·788	1·784	1·784	86°	14·336	14·288	
26°	1·113	1·112	1·107	57°	1·836	1·832	1·832	87°	19·107	19·044	
27°	1·122	1·121	1·116	58°	1·887	1·883	1·883	88°	28·654	28·559	
28°	1·133	1·132	1·127	59°	1·942	1·937	1·938	89°	57·299	57·109	
29°	1·143	1·142	1·137	60°	2·000	1·995	1·997	90°	∞	∞	
30°	1·155	1·154	1·149								

Notes on the Zenithal Projections

Assuming the plane of projection to be the tangent plane at the pole in all these projections the meridians are straight lines radiating at equal angles from the pole and parallels of latitude are concentric circles. The radii of the parallels and the scales along

the meridians and parallels vary in the different projections according to different laws. If L denote the latitude, the radius of the circle on the sphere is $r \cos L$ when r is the radius of the sphere. The length of a degree of latitude is $\frac{\pi r}{180}$ and that of a degree of longitude $\frac{\pi r \cos L}{180}$, neglecting the ellipticity of the earth. The following table gives the corresponding quantities for latitude θ in the Zenithal projections named. As the scale of latitude varies along the meridian the length of a degree in latitude θ must be understood to be n times the length of $\frac{1}{n}$th of a degree in latitude θ, where n is very large. The ratio of this to $\frac{\pi r}{180}$ gives the meridian scale and the ratio of the length of the degree of longitude to $\frac{\pi r \cos L}{180}$ or the ratio of the length of the radius of the parallel to $r \cos L$ gives the scale of longitude. It will be seen that in the stereographic projection both scales are reduced in the same ratio, by dividing the true lengths by $1 + \sin L$. It will also be noticed that in the case of the Zenithal Equal-Area projection the product of the two scales is unity.

TABLE II.

	Radius of parallel.	Degree of longitude.	Degree of latitude.
Gnomonic projection .	$r \cot L$	$\frac{\pi r}{180} \cot L$	$\frac{\pi r}{180 \sin^2 L}$
Stereographic (orthomorphic) projection .	$\frac{r \cos L}{1 + \sin L}$	$\frac{180}{\pi r} \cdot \frac{\cos L}{1 + \sin L}$	$\frac{\pi r}{180} \cdot \frac{1}{1 + \sin L}$
Orthographic projection	$r \cos L$	$\frac{\pi r}{180} \cos L$	$\frac{\pi r}{180} \sin L$
Zenithal Equidistant .	$r\left(\frac{\pi}{2} - L\right)$	$\frac{\pi r}{180}\left(\frac{\pi}{2} - L\right)$	$\frac{\pi r}{180}$
Zenithal Equal Area .	$r\sqrt{2(1 - \sin L)}$	$\frac{\pi r}{180}\sqrt{2(1 - \sin L)}$	$\frac{\pi r}{180} \cdot \frac{\cos L}{\sqrt{2(1 - \sin L)}}$

When the centre of the map is not at the pole, the meridians do not radiate from the centre and the parallels are not concentric circles. To enable the graticule to be drawn, tables are published showing for different central latitudes the azimuth and central distances of the intersections of the meridians and parallels, and by plotting these points and drawing curves through them, the meridians and parallels are laid down.

Note on Mollweide's Projection

The following table gives the distances from the equator of the parallels for each 10° in Mollweide's projection in terms of the radius R of the equivalent sphere. The polar axis of the ellipse is equal to $\sqrt{2}R$, and the equatorial axis to $2\sqrt{2}R$. It will be noticed that the scale along the equator is reduced in the ratio of $2\sqrt{2}$ to π or 2·828 to 3·1416. The meridian scale in the centre of the map is increased in the same proportion.

TABLE III.

Parallel of latitude.	Distance from the equator.
10°	0·194R
20°	0·385R
30°	0·571R
40°	0·751R
50°	0·921R
60°	1·077R
70°	1·219R
80°	1·336R
90°	1·414R

The Length of a Degree

The following table gives the length in miles of a degree of latitude and of a degree of longitude for intervals of 10° of latitude, as shown in the first column, taking into account the ellipticity of the earth. The length of the degree of latitude may be regarded as measured from half a degree below to half a degree above the latitude shown in the first column.

TABLE IV.

At latitude	Length in miles of a degree of	
	Latitude.	Longitude.
0°	68·69	69·15
10°	68·70	68·11
20°	68·77	65·01
30°	68·88	59·94
40°	69·00	53·05
50°	69·10	44·54
60°	69·21	34·66
70°	69·32	23·73
80°	69·38	12·05
90°	69·39	0

The ellipticity of the meridian or the ratio of the difference of the equatorial and polar diameter to the equatorial diameter may be taken to be a little greater than 1/300.

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PRINTED BY GEORGE PHILIP & SON, LTD., LONDON.